A Tale of Two Models:

A confederated approach to unstructured problem solving

R. Hank Donnelly
Benjamin Ashwell

Institute for Defense Analyses
4850 Mark Center Drive • Alexandria, Virginia
22311-1882

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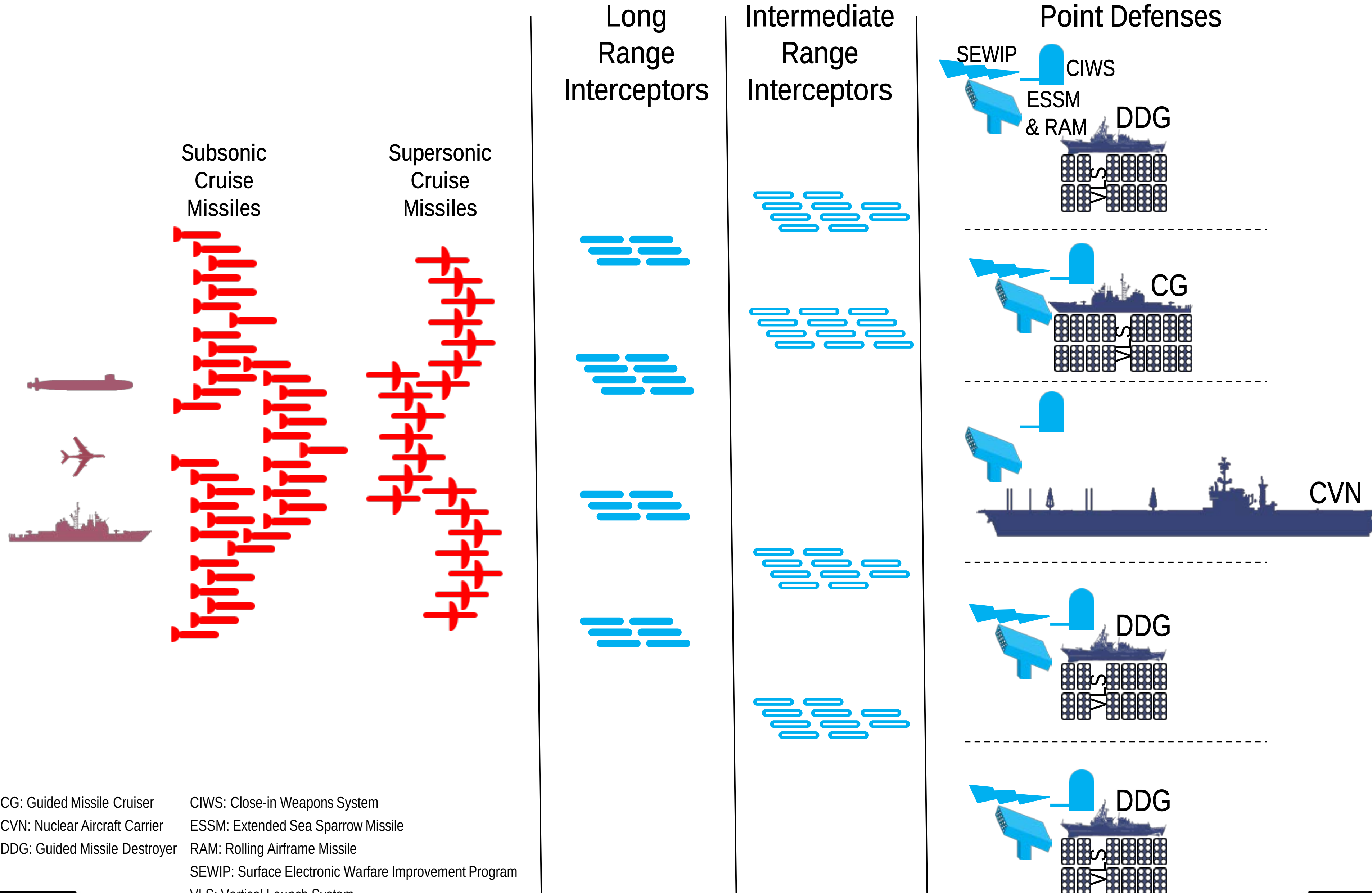
How survivable is an aircraft carrier?



CG: Guided Missile Cruiser
CVN: Nuclear Aircraft Carrier
DDG: Guided Missile Destroyer

CIWS: Close-in Weapons System
ESSM: Extended Sea Sparrow Missile
RAM: Rolling Airframe Missile
SEWIP: Surface Electronic Warfare Improvement Program
VLS: Vertical Launch System

What makes blue lose?

<u>You run out of interceptors</u>

…and get overwhelmed.

<u>You run out of time</u>

…and are hit with unfired interceptors.

When does this occur?

Bring more interceptors

- How many more?
- Which kinds?

Improve your performance

- Which interceptors?
- How much better?

Fire at longer ranges

- How much farther?
- Which interceptors?

Fire faster

- How much faster?
- How coordinated?

Get other systems to help

- What capabilities?
- How many?

Complementary approaches

Closed-form analytic

- Compare many different excursions
- Understand the nature of the trade space

Numerical

- Expose the effects from system-system interactions
- Understand the quirky impact of reality

Closed-form analytic approach

Expected number of interceptions depends on fire doctrine:

Shoot-Look-Shoot (SLS): $\langle I \rangle = \frac{n_x p_x}{2} + \frac{n_x p_x}{2} = n_x p_x$

SSLSS: $\langle I \rangle = \frac{n_x\left(1-(1-p_x)^2\right)}{4} + \frac{n_x\left(1-(1-p_x)^2\right)}{4} = n_x p_x \left(1 - \frac{p_x}{2}\right)$

SLSS: $\langle I \rangle = \ldots = n_x\, p_x \frac{\left(3-3p_x+{p_x}^2\right)}{(3-2p_x)}$

Complex: $\langle I \rangle = \frac{n_y}{2}\left[1 - (1-p_x)^2\left(1-p_y\right)^2\right] + \left[\left(n_x - n_y\right)\right] p_x \left(1 - \frac{p_x}{2}\right)$

(e.g., two kinds of missiles fired as pairs of pairs

$n_x > n_y$, and both are even)

n=# of interceptors
p=probability of kill
f=fraction of inventory

Closed-form analytic approach

Which way is "uphill"… in nine dimensions?

Or, which component(s) produce the greatest increase in the expected number of intercepts?

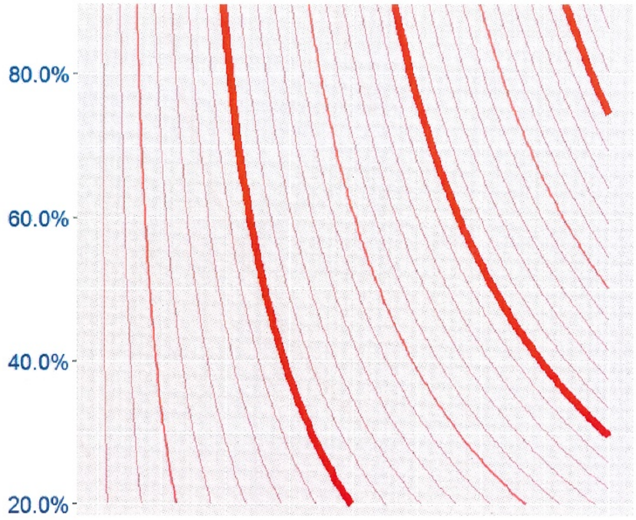
$$\nabla I(n_w, n_x, n_y, n_z, p_w, p_x, p_y, p_z)$$

$$= \frac{\partial I}{\partial n_w}\widehat{\boldsymbol{n_w}} + \frac{\partial I}{\partial n_x}\widehat{\boldsymbol{n_x}} + \frac{\partial I}{\partial n_y}\widehat{\boldsymbol{n_y}} + \frac{\partial I}{\partial n_z}\widehat{\boldsymbol{n_z}} + \frac{\partial I}{\partial p_w}\widehat{\boldsymbol{p_w}} + \frac{\partial I}{\partial p_x}\widehat{\boldsymbol{p_x}} + \frac{\partial I}{\partial p_y}\widehat{\boldsymbol{p_y}} + \frac{\partial I}{\partial p_z}\widehat{\boldsymbol{p_z}}$$

$$= p_w\widehat{\boldsymbol{n_w}} + p_x\left(1 - \frac{p_x}{2}\right)\widehat{\boldsymbol{n_x}} + p_y(1-p_x)^2\left(1 - \frac{p_y}{2}\right)\widehat{\boldsymbol{n_y}} + \frac{p_z(3 - 3p_z + {p_z}^2)}{(3-2p_z)}\widehat{\boldsymbol{n_z}}$$

$$+ n_w\widehat{\boldsymbol{p_w}} + (1-p_x)\left(n_x - 2n_yp_y + n_y{p_y}^2\right)\widehat{\boldsymbol{p_x}}$$

$$+ n_y(1-p_x)^{\mathbf{2}}(1-p_y)\widehat{\boldsymbol{p_y}} + \frac{n_z(9 - 18p_z + 15{p_z}^2 - 4{p_z}^3)}{(3-2p_z)^2}\widehat{\boldsymbol{p_z}}$$



Closed-form analytic approach

$$\nabla I\left(n, f_D, f_w, f_x, f_y, f_z, p_w, p_x, p_y, p_z\right)$$

$$\nabla I(100, 0.5, 0.42, 0.16, 0.05, 0.42, 0.5, 0.5, 0.5, 0.5)$$ [fictional data]

Contributions to improvement with a 10% relative increase:

- Overall number of missiles: 44%
- Fraction dedicated to defensive missiles: 44%
- Fraction of defensive of type w (SLS): 0.08%
- Fraction of type x (larger of pair of pairs): 3.16%
- Fraction of type y (smaller of pair of pairs): 0.15%
- Fraction of type z (dependent on other types)
- Effectiveness of type w: 4.82%
- Effectiveness of type x: 0.79%
- Effectiveness of type y: 0.07%
- Effectiveness of type z: 3.18%

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“Hunchback” is a simulation written in python

You can think of it as a board game

Each blue surface combatant follows a decision tree, and the rest is bookkeeping – keeping track of the dozens or hundreds of ASCMs and interceptors in flight

The ultimate goal is to roughly approximate the doctrines and timelines of real engagements, which lets us compare and contrast the impact of changes we might make

The realism of the simulation comes from combining performance data with limitations on blue's ability to engage threats

Interceptors have min/max ranges and fixed speeds

Blue cannot launch its entire inventory simultaneously

Blue cannot shoot things it cannot see

Closely grouped threats present additional challenges

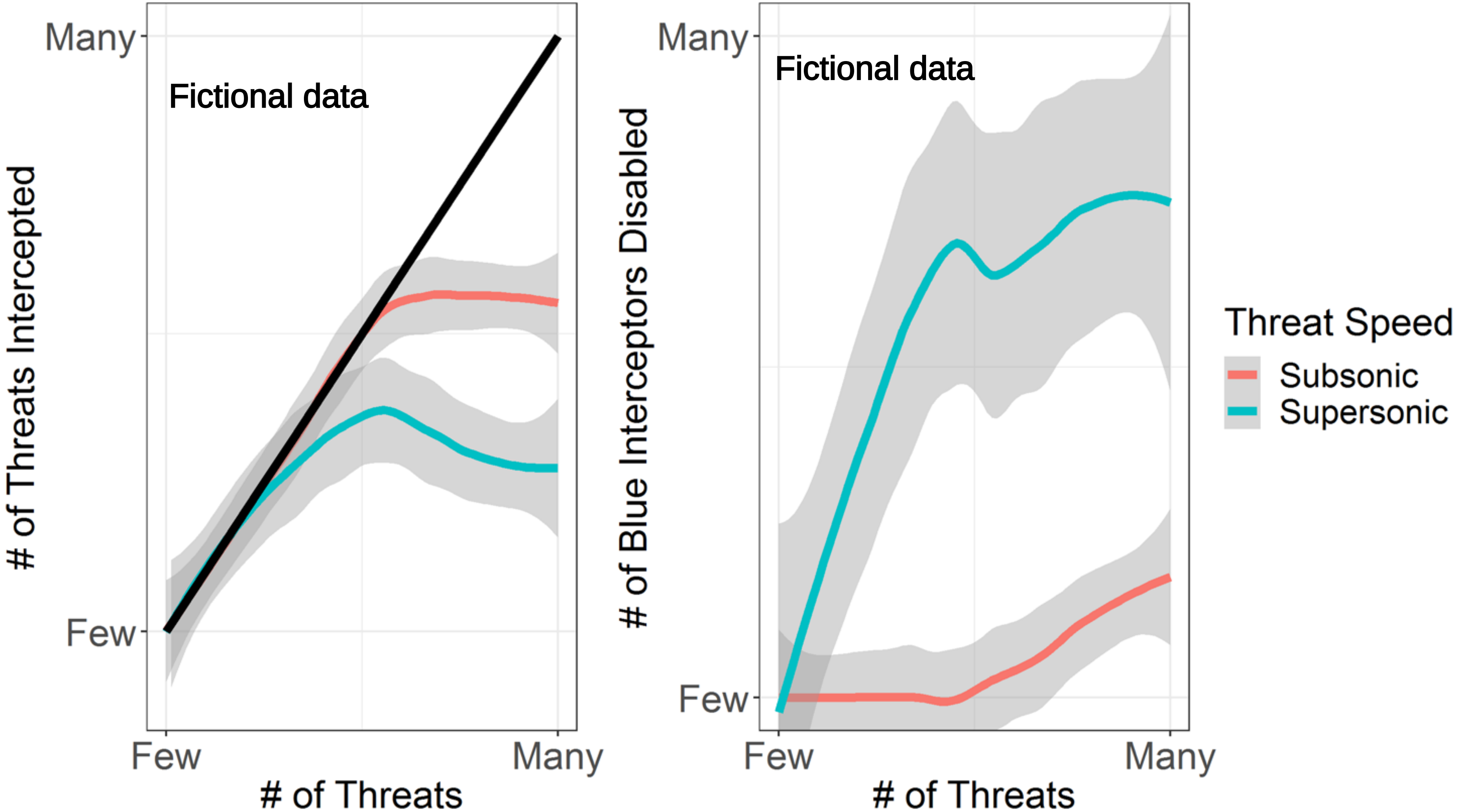
Blue ships must coordinate with one another

Red's coordination influences blue's reaction time

Blue ships can sink with unfired interceptors

Hunchback does **not** model blue offensive capabilities (and should not!)

Threat speed causes a qualitative difference in performance

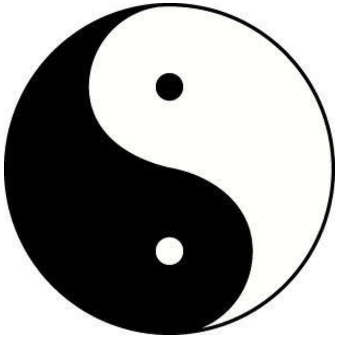


All models are wrong, but some are useful

Confirm agreement between models

Using the utility of one model to fill in the gaps present in another

- Numerical model:
  - Provides detailed and nuanced understanding of complex system
  - Computationally expensive, have to limit sampling locations



- Closed-form analytical:
  - Provides visibility into broad functional behaviors and trends
  - Requires simplifying assumptions, nuances, and details lost

Although not an exact calculation of the real world performance, allows for a compare and contrast of investment choices

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