Power Calculations for Categorical Factors

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Power Calculations for the Real World

- We've seen some of the "messy" cases, but what about the simpler case of linear regression?
  - minimum effect size to be detected
  - significance level of the hypothesis test
  - variability
  - design characteristics
- For two-level factors, the effect size is directly related to the coefficients in a least squares regression model - easy to calculate power
- We'll use Δ, max-min over design range
  - Δ=2 corresponds to coefficient of 1

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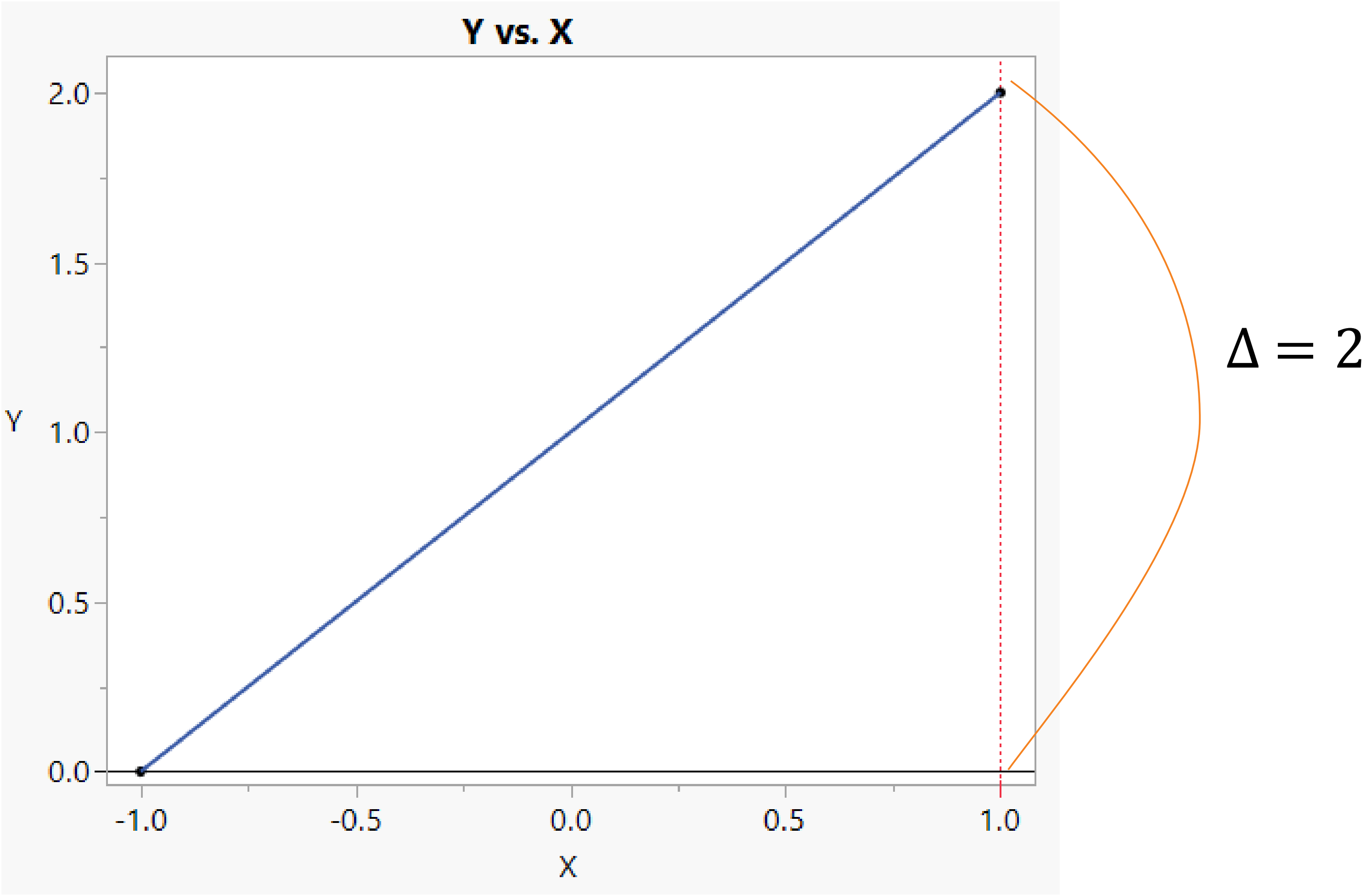
Continuous Factor

Difference between max and min from low to high settings of X



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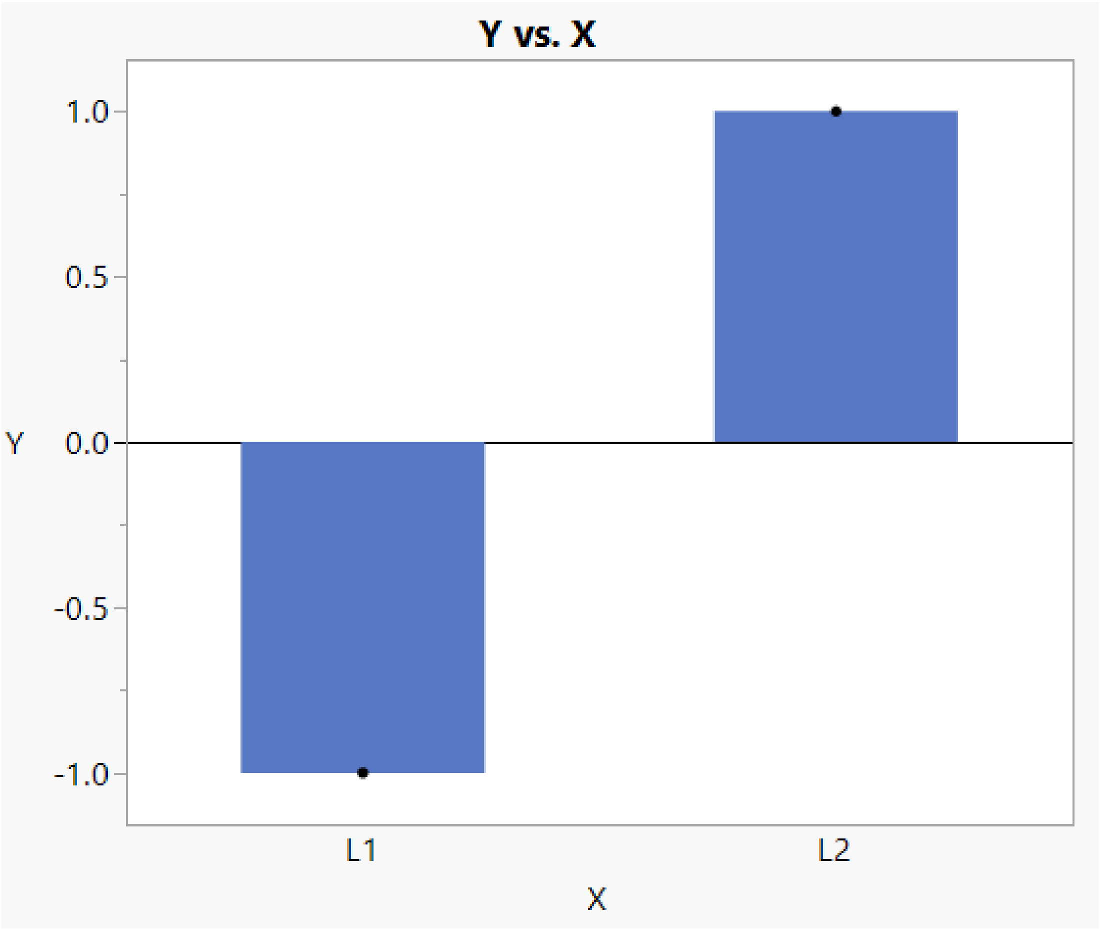
2-level Categorical Factor

Difference between max and min from the two different levels of X



$\Delta = 2$

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Power for Categorical Factors

For number of levels: *k > 2*

Usually interested in the overall effect rather than individual parameters.

H_0: All treatments (levels) have the same effect

H_a: Not all treatment effects are the same

"Effect Tests" in Fit Model or

"Effect Power" in Custom Design

What does Δ mean with $k > 2$ levels?

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Notes

- Error/RMSE/sigma = 1
  - This way we only need to talk about Δ in terms of max-min.
- Fix α=0.05
- Categorical effects will use the “sum to zero” constraint.
  - For *k* levels, only specify k-1 coefficients.

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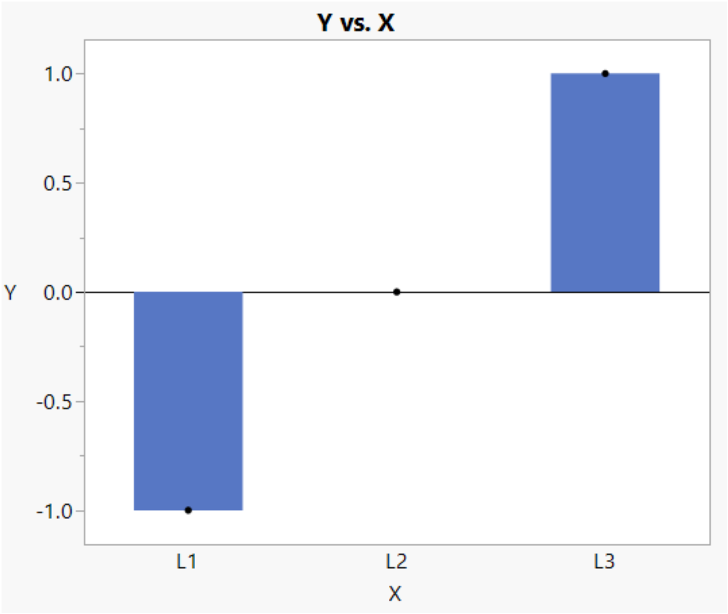
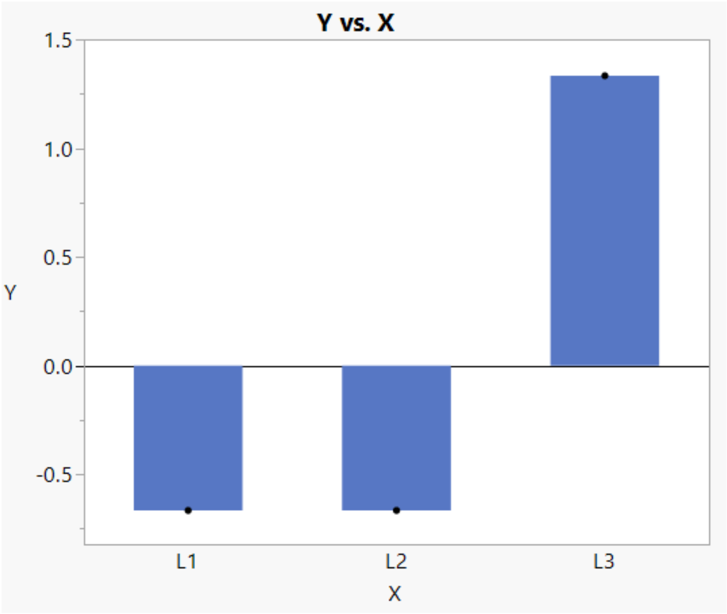
$k = 3$

Which is the best choice for $\Delta = 2$?





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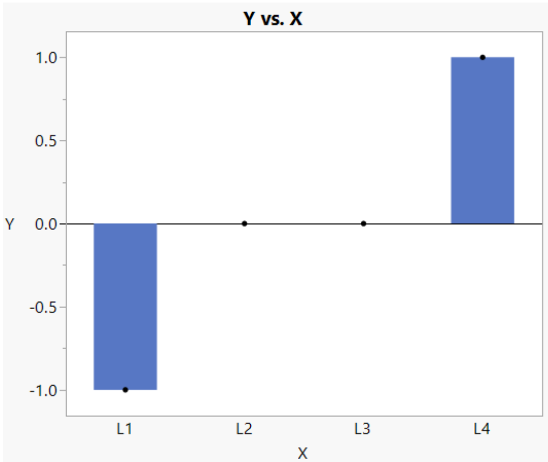
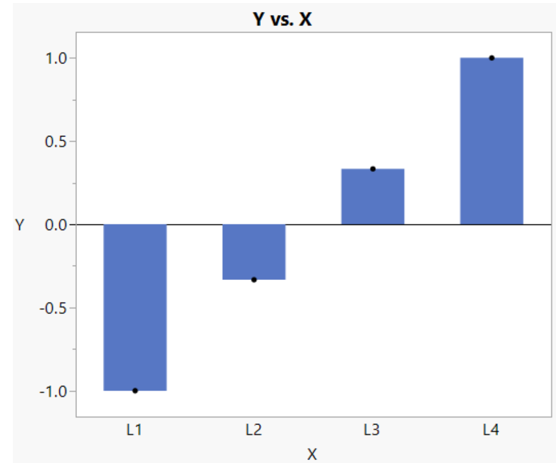
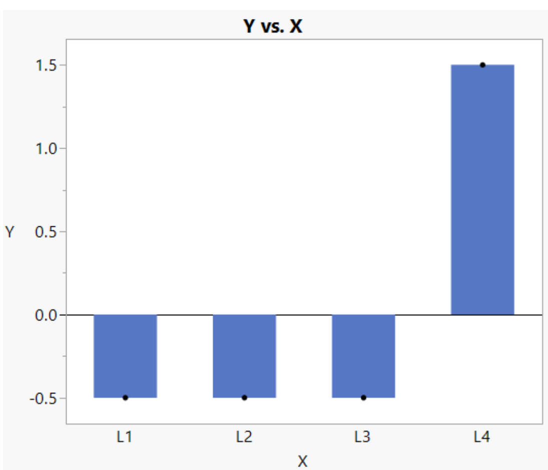
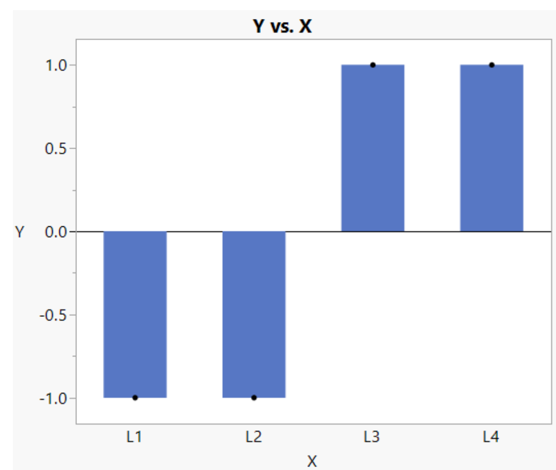
$k = 4, \Delta = 2$









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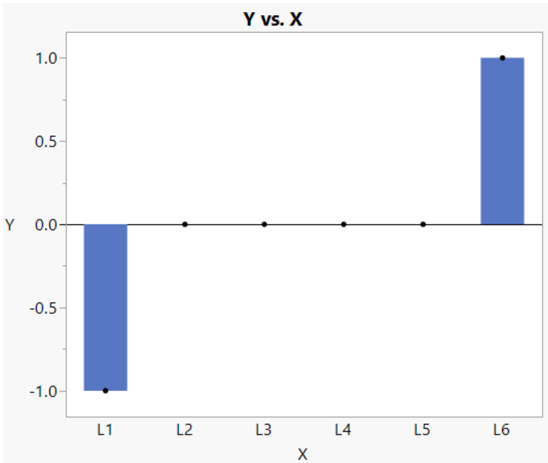
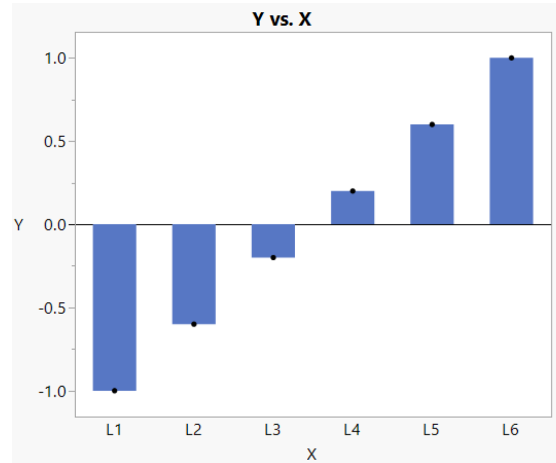
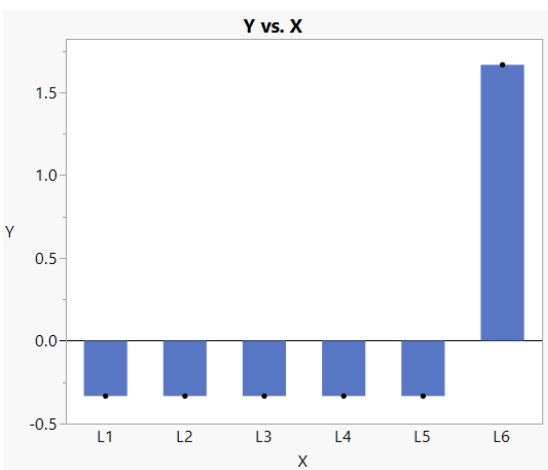
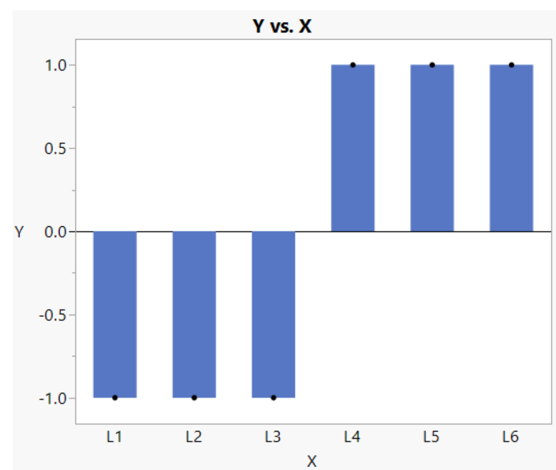
$k = 6, \Delta = 2$









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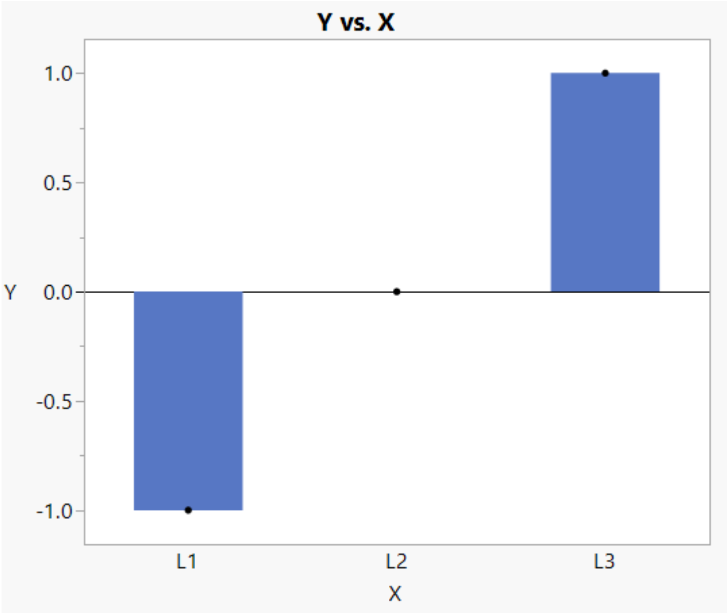
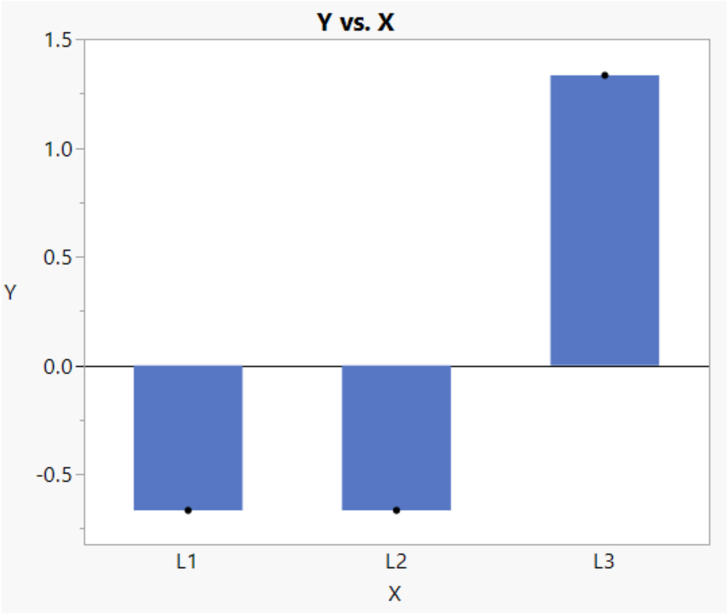
$k = 3, N = 24, \Delta = 2$



power = .924



power = .977

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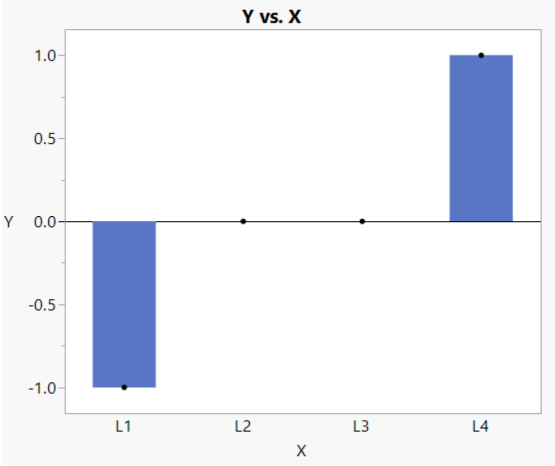
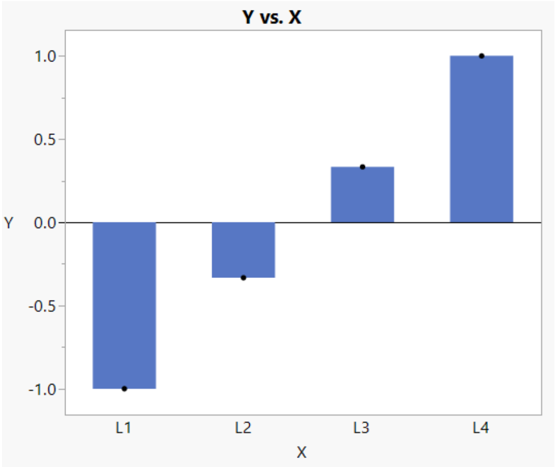
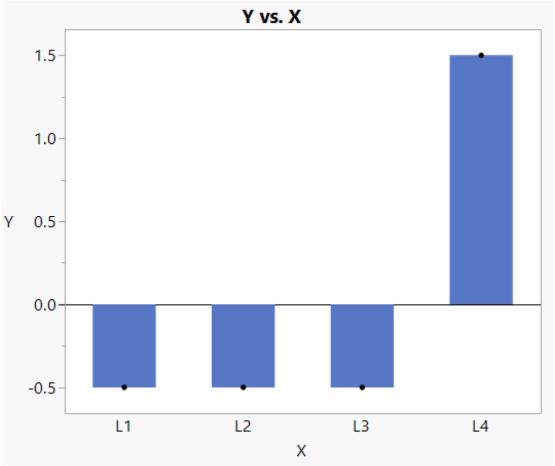
$k = 4, N{=}24, \Delta = 2$



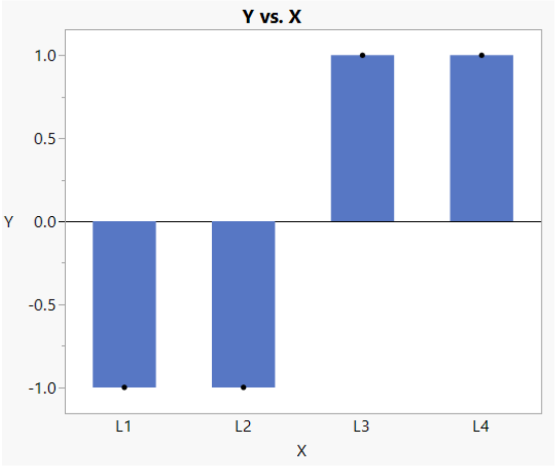
power = .755



power = .802



power = .912



power = .973

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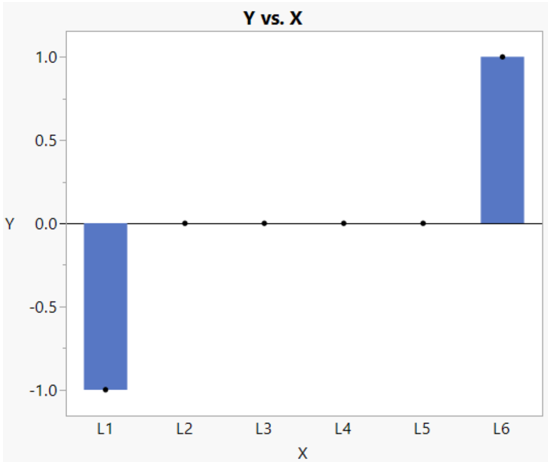
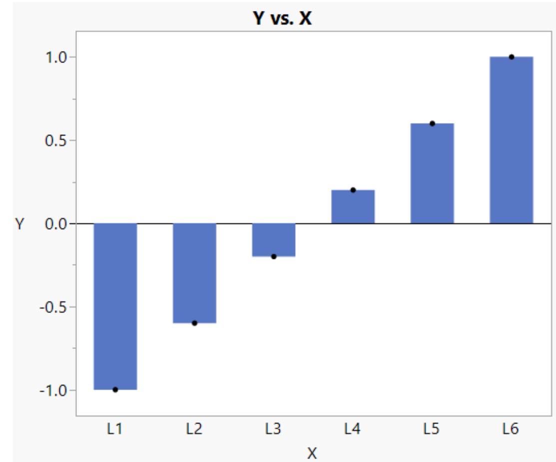
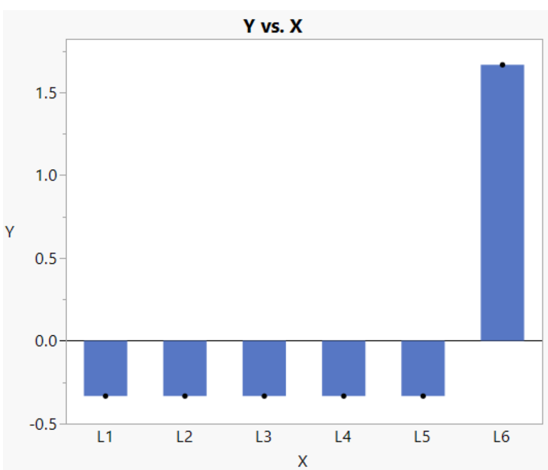
$k = 6, N=24, \Delta = 2$



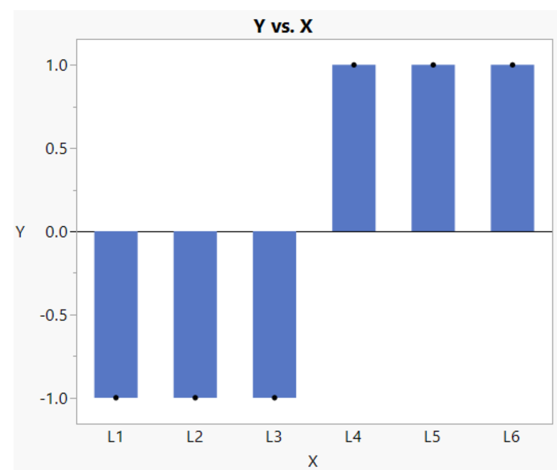
power = .432



power = .586



power = .674



power = .923

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Why does it matter?

Consequences of using the worst-case

- Use additional runs to achieve desired power.
- Drop levels out of consideration from a categorical factor.
- May not run the experiment at all.

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What does $\Delta = 2$ mean for $k>2$?

- Does the worst-case seem plausible?
- There are infinitely many choices once we fix the max and min.
- What if we think about simulating possible configurations for the effects?

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Treatment effects

- Let $\boldsymbol{\tau}^a = (\tau_1^a, \tau_2^a, \ldots, \tau_k^a)'$ be vector of treatment effects under H_a.
- Set

$$\tau_k^a - \tau_1^a = \Delta$$

$$\tau_1^a \le \tau_2^a \le \ldots \le \tau_k^a$$

- Assign multivariate distribution $f_\tau(\cdot)$ with domain $[\tau_1^a, \tau_k^a]$.
- Reflects the belief of the effects in $\boldsymbol{\tau}^a$.

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Sum-to-zero constraint

- Easy to modify the vector of treatment effects.
- Let $\boldsymbol{\tau}^{a*}$ be the constrained vector, generated from

$$(\tau_1^a - \delta, \tau_2^a - \delta, \ldots, \tau_i^a - \delta, \ldots, \tau_k^a - \delta)'$$

where $\delta = \sum_i \tau_i^a / k$

- Then we can generate the vector and adjust.

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Power Function

$$\eta(D, \boldsymbol{\tau}^a) = P(\text{reject } H_0 | H_a : \boldsymbol{\tau} = \tau^{a*} \text{ is true})$$

- Random variable for design *D,* under domain of $f_\tau(\cdot)$.
- Can simulate from $f_\tau(\cdot)$ to assess the power.

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Benefits of the Power Function

Allows us to ask questions

- What is the average power?

$$E[\eta(D, \boldsymbol{\tau}^a)] = \int \eta(D, \boldsymbol{\tau}^a) f_\tau(\boldsymbol{\tau}^a) \mathrm{d}\boldsymbol{\tau}^a.$$

- What is the probability that the power is at least 80%?
- What is the .05 quantile of the power distribution?
  - 95% of the time the power is greater than what number?

Off to JMP…

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Key Takeaways

- Power for categorical factors is not so simple.
- Only looking to the worst-case can be costly and not a practically likely situation.
- We can use the idea of simulating the treatment effects to get a distribution of the power.
- Think about power for how you do the analysis.

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