Applying parametric survival models to analyze target location error (TLE) was the brainchild of Todd Remund, Greg Hutto, and Jeff Beekman (Beekster).

This presentation details how the parametric survival model was adapted to TLE.

An example is given- input file and source code in Python are available
Introduction

• Basic idea: precision target location, germane to navigation, weapon delivery and target tracking, can be related to distance from a target, as well as other possible explanatory factors (elevation and azimuth angles to the target, for example)

• Use the approach presented in Meeker and Escobar, *Statistical Reliability*, to use a generalized linear model based on log-link distribution function.

• Model fitting capabilities exist in JMP, as well as code written in Python and R
CEP

• CEP, CE10 and CE90 defined as
  • radius of a circle about the target that has the property that 50%, 10%, or 90% (respectively) of the values are within a circle of radius ‘CEP’ and so forth

• Using parametric survival model, estimate CEP (or CE90, CE10) as functions of range R and associated covariates

• Additionally, we’d like confidence intervals for CE estimates, an RMS error estimate and 95/90 tolerance limits
Development

• TLE error estimates are similar to reliability of a component;
  • The cumulative probability of failure can be related to time in service
  • Similarly, cumulative probability of TLE can be related to range to the target
• Reliability (parametric) modeling provides a way to specify CEP (or CEwhatever) as a function of range to target.
Parametric Model

• Summary: generalized linear model

• \( P(T \leq t_i | X) = \Phi\left(\frac{\log(t_i) - \mu_i}{\sigma_i}\right) \), where
  \( \mu_i = b_0 + b_1 X_i \) and \( \sigma_i = b_2 + b_3 X_i \)

• Percentile estimates, RMS, tolerance intervals all given by \( \Phi \)

• \( X \) an \( n \times m \) design matrix, columns contain covariates

• Details in Meeker and Escobar, *Statistical Reliability*
A few details

• $X$, design matrix, has $n$ rows, $m$ columns, $X(i,j) = i$-th observation, explanatory variable $j$

• In terms of matrix algebra:
  • $\mu = X\beta + \epsilon$,
  • $\mu$ is $n \times 1$, $X$ is $n \times m$, $\beta$ is $m \times 1$, and $\epsilon$ is $n \times 1$
  (similarly for $\sigma$)

• Estimates of $\beta$ found by maximum likelihood

• Log link functions:

<table>
<thead>
<tr>
<th>Log Error</th>
<th>Link Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log normal</td>
<td>normal</td>
</tr>
<tr>
<td>Extreme value</td>
<td>Weibull</td>
</tr>
<tr>
<td>log-logistic</td>
<td>logistic</td>
</tr>
</tbody>
</table>
Link Equations

• The key is the link between the distribution of the log of a random variable and the distribution of the random variable;
  • $Y \sim$ Weibull, then $\log(Y) \sim$ gumbel (EVS)
  • $Y \sim$ Normal, then $\log(Y) \sim$ lognormal
  • $Y \sim$ Logistic, then $\log(Y) \sim$ loglogistic

• Fit a generalized linear model to the $\log(Y)$- in the ‘normal’ case:

\[ P(\log(Y) < y) = \Phi_{\ln} \left( \frac{y - \mu_r}{\sigma_r} \right), \]

$\mu_r, \text{ and } \sigma_r$ functions of covariates-
Estimation

• Estimation of parameters, $\beta_0$, $\beta_1$, and $\sigma$ is based on the likelihood function

$$likelihood(\beta_0, \beta_1, \sigma) = \prod_{i=1}^{n} \frac{1}{\sigma} \phi\left(\frac{y_i - \mu_i}{\sigma}\right)$$

• $\mu_i = \beta_0 + \beta_1 X_i$ and $\sigma_i = \beta_2 + \beta_3 X_i$

• Maximum likelihood estimates of $\beta'$s
Normal-lognormal case

For $\Phi_{\ln}$, the log normal distribution function,

$$\Pr(Y \leq y) = F(y; \mu, \sigma) = F(y; \beta_0, \beta_1, \sigma) = \Phi_{\ln}((y - \mu) / \sigma)$$

The quantile function for this model is

$$y_p(r) = \mu + \Phi_{\ln}^{-1}(p)\sigma = \beta_0 + \beta_1 r + \Phi_{\ln}^{-1}(p)\sigma$$

The quantile function then gives us probability curves for TLE, just select ‘p’
Upshot..

- Parametric modeling, if appropriate may avoid the effort to get IID errors
- Drawback- modeling assumes each realization (data run) is representative of a stationary, ergodic process
- Better approach may be a hierarchical model
Example: Original data
Weibull distribution
Questions??

**Panel 1:**
We’re looking for engineers with short telomeres for their age.

**Panel 2:**
That’s an indication that you value work above exercise.

**Panel 3:**
But you have a company gym. That’s our slacker trap!