Software Reliability Modeling: Heterogeneous Fault Detection Processes

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Outline

• Motivation
• Background
• Contribution
• NHPP SRGM with single changepoint
  – Homogeneous and heterogeneous changepoint models
  – Illustrations
• Conclusions and future work
Motivation

• Software reliability growth models (SRGM)
  – Well-established methodology
  – Many based on non-homogeneous Poisson process (NHPP)
    • Expected number of faults if debugging performed indefinitely
    • Failure intensity
    • Mean time to failure (MTTF)
    • Reliability

• SRGMs assume
  – Fault detection rate dependent on software testing time
Background

• During software testing
  – Failure detection affected by many factors
    • Change in testing environment
    • Testing strategy
    • Integration testing
    • Resource allocation

• Impact of factors on the fault detection process – Changepoints
  – Failing to model changepoints may adversely affect system assessment

• Several NHPP SRGM consider changepoint since 1992
  – Consider only homogeneous combinations of failure distribution before and after changepoints
Contribution

• Heterogeneous single changepoint models
  – Applies ECM algorithm to maximize log-likelihood

• Compared with existing homogeneous models
  – Demonstrate heterogeneous changepoint models characterize some data sets better than homogeneous models
NHPP SRGM

• Stochastic process
  – Counts number of events observed as function of time
  – In software reliability,
    • counts number of faults detected by time $t$

• Counting process characterized by mean value function (MVF)
  – Form of MVF of several SRGM:
    $$ m(t) = a \times F(t) $$
    • $a$ – expected number of faults detected with indefinite testing
    • $F(t)$ - cumulative distribution function (CDF)
NHPP SRGM

- Substituting exponential distribution for $F(t)$
  \[ m(t) = a(1 - e^{-bt}) \]
  - $b$ - fault detection rate
  - Also known as Goel-Okumoto (GO) SRGM

- Substituting delayed S-shaped (DSS) distribution for $F(t)$
  \[ m(t) = a(1 - (1 + bt)e^{-bt}) \]
  - $bte^{-bt}$ – can characterize delay between time of failure observation and reporting or fault masking
NHPP SRGM with single Changepoint

• $T = \langle t_1, t_2, \ldots, t_n \rangle$ - vector of failure time data with single changepoint

• $F_1(t)$ and $F_2(t)$ - failure distributions before and after $\tau$

• MVF of NHPP SRGM with single changepoint

$$m(t) = \begin{cases} a \times F_1(t), & 0 \leq t \leq \tau \\ a(F_1(\tau) + F_2(t - \tau)), & t > \tau \end{cases}$$
NHPP SRGM w/single Changepoint (2)

- NHPP SRGM with single changepoint can be
  - Homogeneous
    - If \( F_1(t) \) and \( F_2(t) \) follow similar distribution
  - Heterogeneous
    - If \( F_1(t) \) and \( F_2(t) \) follow different distribution

- Changepoint \( \tau \) identified by maximizing likelihood for each value \( \tau \in (2, (n - 1)) \)
Homogenous changepoint models

1. GO-GO SRGM
   - If $F_1(t)$ and $F_2(t)$ follow **exponential** distribution
     \[ m(t) = \begin{cases} 
     a(1 - e^{-bt}), & 0 \leq t \leq \tau \\
     a \left( (1 - e^{-b\tau}) + (1 - e^{-b(t-\tau)}) \right), & t > \tau 
     \end{cases} \]

2. DSS-DSS SRGM
   - If $F_1(t)$ and $F_2(t)$ follow **S-shaped** distribution
     \[ m(t) = \begin{cases} 
     a \left( 1 - (1 + bt) e^{-bt} \right), & 0 \leq t \leq \tau \\
     a \left( (1 - (1 + b\tau) e^{-b\tau} + (1 + b(t - \tau)) e^{-b(t-\tau)}) \right), & t > \tau 
     \end{cases} \]
Heterogenous changepoint models (2)

1. GO-DSS SRGM
   - If $F_1(t)$ and $F_2(t)$ follows exponential and S-shaped distribution respectively
   
   $$m(t) = \begin{cases} 
   a(1 - e^{-bt}), & 0 \leq t \leq \tau \\
   a \left( (1 - e^{-b\tau}) + (1 + b(t - \tau))e^{-b(t-\tau)} \right), & t > \tau 
   \end{cases}$$

2. DSS-GO SRGM
   - If $F_1(t)$ and $F_2(t)$ follows S-shaped and exponential distribution respectively
   
   $$m(t) = \begin{cases} 
   a(1 - (1 + bt)e^{-bt}), & 0 \leq t \leq \tau \\
   a \left( (1 - (1 + b\tau)e^{-b\tau}) + (1 - e^{-b(t-\tau)}) \right), & t > \tau 
   \end{cases}$$
Parameter Estimation Method

- Maximum likelihood estimation (MLE) maximizes the likelihood function to identify numerical values of model parameters.

- NHPP failure times data log-likelihood:
  \[ LL(\Theta|T) = -m(t_n) + \sum_{i=1}^{n} \log(\lambda(t_i)) \]
  - \( \Theta \) – vector of model parameters
  - \( \lambda(t) := \frac{dm(t)}{dt} \) - instantaneous failure rate at time \( t \)

- ECM algorithm applied to identify MLEs.
Initial parameter estimates selection

- Function of $f(t_i; \Theta)$

\[ a^{(0)} = N \]

and

\[ \Theta^{(0)} := \sum_{i=1}^{n} \frac{\partial}{\partial \Theta} \log[f(t_i; \Theta)] = 0 \]
Initial parameter estimates selection (2)

- GO SRGM parameters

\[ a^{(0)} = n, \quad b^{(0)} = \frac{n}{\sum_{i=1}^{n} t_i} \]

- Similarly, for DSS SRGM

\[ b^{(0)} = \frac{2n}{\sum_{i=1}^{n} t_i} \]
Illustrations
GO-GO SRGM applied to SYS1 dataset

Changepoint at $\tau = 16$, $(t_{\tau=16} = 1,056)$ maximizes likelihood
GO SRGM with and without CP

GO with changepoint fits data better
GO SRGM with and without CP (2)

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<th>1-CP GO AIC</th>
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GO with changepoint significantly better on 9 of 10 datasets
## DSS SRGM with and without CP

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DSS with changepoint significantly better than model without
DSS SRGM with and without CP (2)

DSS with changepoint fits S27 data better
Homogeneous vs. Heterogeneous models

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<th>GO-DSS</th>
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Heterogeneous models preferred in six out of ten data sets
Summary, Conclusion, and Future work
Summary and Conclusion

• Developed heterogeneous single changepoint model
• Assesses models with and without changepoint
  – Models with changepoint often characterizes data better
• Compared homogeneous vs. heterogeneous models
  – Heterogeneous models outperformed homogeneous models on 60% of datasets considered
Future work

• Theoretical
  – Model selection procedure considering additional combinations of possible models
  – Performance optimization of algorithmic approach

• Empirical
  – Methods to assess the effectiveness of milestone decisions and testing between milestones
  – Combine with metrics-based models to identify effective activities
Acknowledgement

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