Operations Research Models for Resource Allocation in Reliability Growth Testing

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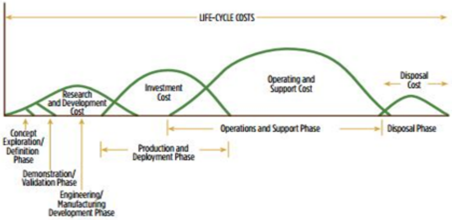
Motivation

- Product development and testing is often resource intensive



- Desirable characteristics of newly fielded system
  - Meets user requirements
  - Performs as specified
  - Is reliable

Research Overview

Setting: Developing test plans for a complex system of systems

How much testing should be completed. . .

- at the system level?
- at the subsystem level?
- on components within a subsystem?

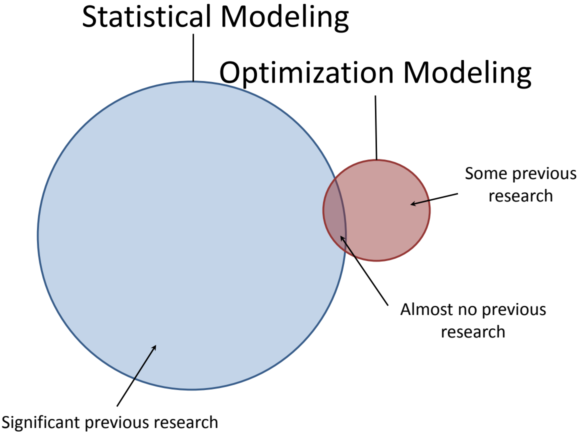
How should testing resources be divided within each level?

Methodology: Apply mathematical optimization to model and solve these resource allocation problems

Reliability Growth Preliminaries

- **Reliability Growth** is the improvement in reliability that occurs when failure modes are detected and removed during developmental testing
- **Existing Research**
    - **Statistical models** to estimate and/or predict reliability growth of a component (or system) using failure data
    - **Optimization models** to allocate limited testing resources across a system of systems

Summary of Existing Reliability Growth Research



Statistical Modeling

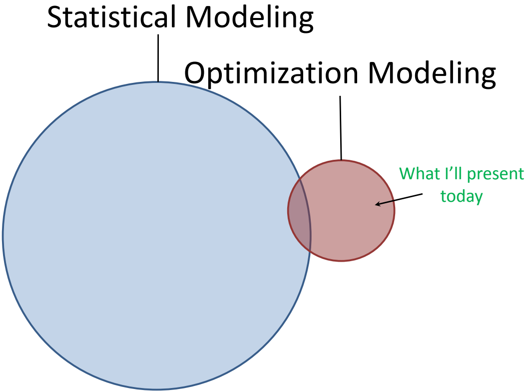
Optimization Modeling

Some previous research

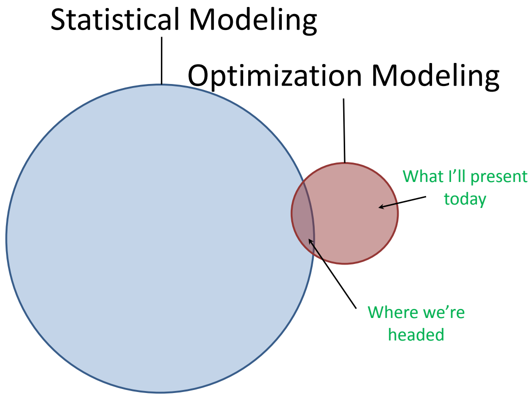
Almost no previous research

Significant previous research

Summary of Existing Reliability Growth Research



Summary of Existing Reliability Growth Research



Statistical Modeling

Optimization Modeling

What I'll present today

Where we're headed

Presentation Outline

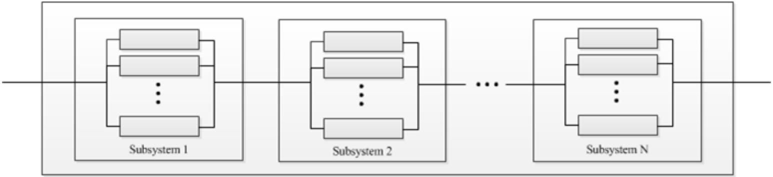
1. Demonstration of Base Resource Allocation Model
2. Demonstration of Multi-Level Testing Model
3. Demonstration of Model with Uncertainty
4. Summarize Ongoing Research Activities

Presentation Outline

1. Demonstration of Base Resource Allocation Model
    - Follows closely related models of Coit (1998); Levitin (2002); Dai et al. (2002)
2. Demonstration of Multi-Level Testing Model
3. Demonstration of Model with Uncertainty
4. Summarize Ongoing Research Activities

Base Model: Assumptions

- System configuration: **Series-parallel** with independent components



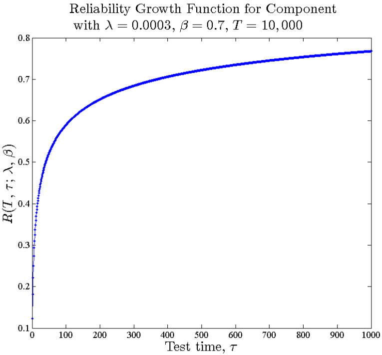
- We must decide how much testing to apply to each design in order to maximize system reliability

Base Model: Assumptions

- After design n is tested for τ_n hours, components of design n will have failure rate
$$u(\tau_n;\lambda_n,\beta_n) = \lambda_n \beta_n \tau_n^{\beta_n - 1}$$
[*AMSAA growth with parameters* $\lambda_n > 0$ *and* $0 < \beta_n < 1$ *(Crow, 1974)*]
- Components are assumed to have constant failure rate after testing is completed
- Design n's after-testing reliability for a mission of length $T > 0$ is given by
$$R(T,\tau_n;\lambda_n,\beta_n) = e^{-u(\tau_n;\lambda_n,\beta_n)T} = e^{-\lambda_n \beta_n \tau_n^{\beta_n - 1} T}$$

Component Mission Reliability vs. Test Time

- Plot of $R(T, \tau_n; \lambda_n, \beta_n)$ for $T = 10,000$



The optimization models and much of the underlying solution methdology presented can be adapted for growth functions that take a different mathematical form.

Illustration of Base Optimization Model

Series Parallel System

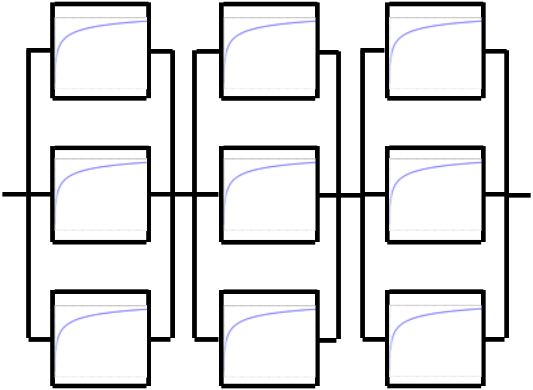
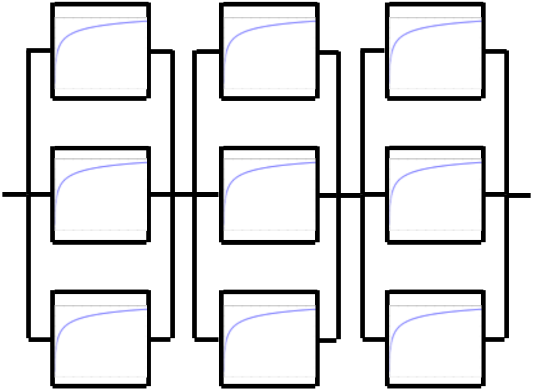


Illustration of Base Optimization Model

Series Parallel System



Components in a subsystem may have same design or different designs

Notation

Indices and Sets

- N_i: Set of component designs in subsystem $i = 1, \ldots, m$
- N: Set of all component designs (i.e., $N = N_1 \cup N_2 \cup \cdots \cup N_m$)

Parameters

- K_n: Redundancy level of design n, $n \in N$
- T: Mission length
- B: Available testing time

Variables

- τ_n: Testing time for design $n \in N$

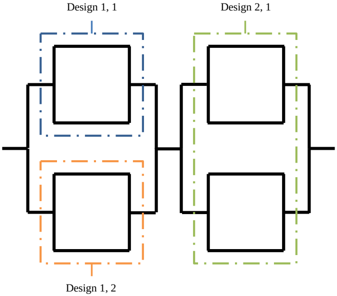
Model Formulation

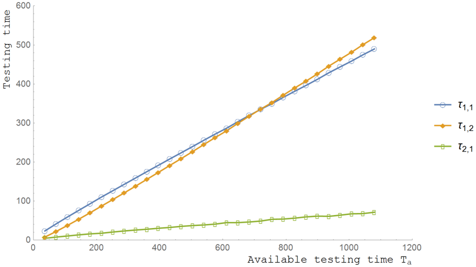
$$f^{\star} \equiv \max \prod_{i=1}^{m} \left[1 - \prod_{n \in N_i} \left(1 - R(T, \tau_n; \lambda_n, \beta_n)\right)^{K_n} \right] \tag{1a}$$

$$\text{s.t.} \sum_{n \in N} \tau_n \leq B \tag{1b}$$

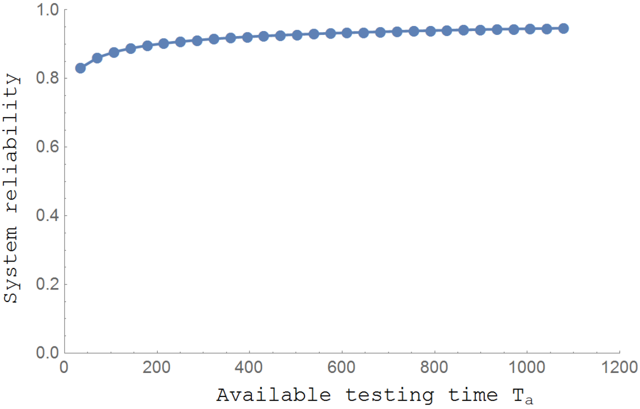
$$\varepsilon \leq \tau_n \leq B, \ \forall n \in N \tag{1c}$$

Demonstration for Notional Instance





Demonstration for Notional Instance



Solution Insights

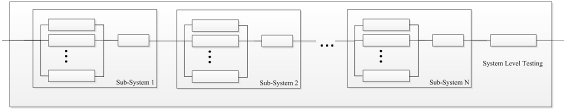
- Designs exhibiting faster growth could be tested more…
- …or less

- Other factors (e.g., redundancy) play a role, too
- Provides a systematic way of analyzing these tradeoffs

Multi-Level Testing

Concept: Testing can be performed at the system, subsystem, or component level

- **System-level testing** is comprehensive, but expensive.
- **Component-level testing** is cheaper, but it may not induce failure modes due to integration



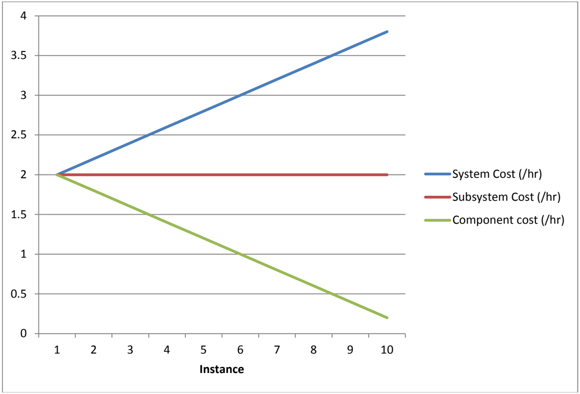
Modifications to Model:

- Integration-related failures now included in model
- Higher-level testing now improves the reliability associated with all integrated sub-components
- Costs are associated with the different testing activities, and the total budget for testing is limited

Demonstration on Notional Instance

Instance setup: 3 subsystems, 3 components per subsystem

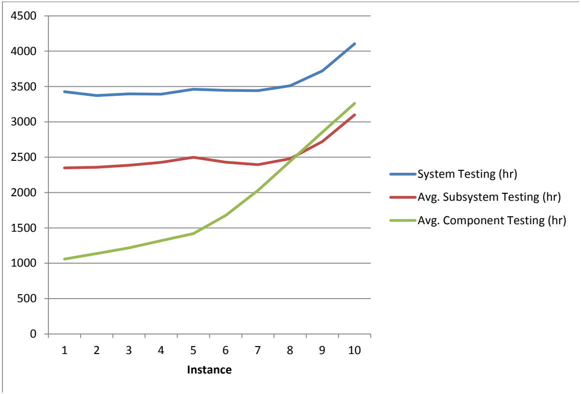
Setup: Testing Cost for 10 Instances



Demonstration on Notional Instance

Instance setup: 3 subsystems, 3 components per subsystem

Results: Testing Times for 10 Instances



Test Planning Under Uncertain Reliability Growth

Concept: Considering uncertainty in growth model parameters will lead to higher quality decisions.

Modeling Approach: Incorporate uncertainty in growth model parameters via using a bilevel optimization approach.

- Which allocation of test times τ maximizes the resulting reliability for some worst-case realization of growth model parameters (λ, β) over some *uncertainty set*?

Summary of Results for Notional Instance

- Process
    - Simulate underlying growth process using test plans generated by either (i) robust/uncertain optimization or (ii) base/deterministic optimization approach
    - Use results from a preliminary round of testing to estimate growth model parameters and/or construct uncertainty sets
- Any “reasonable” uncertainty set construction leads to a statistically significant improvement in reliablity

Summary and Future Work

Summary:

- Developed models, which draw from existing reliability growth models, for allocating testing resources within a system of systems
- Extended the model to demonstrate additional benefits by incorporating testing at multiple levels (system, subsystem, etc.) and incorporating parameter uncertainty in the model

Future Work:

- (Ongoing) Sequential testing in a Bayesian environment
- (Ongoing) Allocating accelerated testing resources
- Dependent components
- Alternative redundancy configurations

Thank you!

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