Always a Step Ahead

UNPARALLELED
COMMITMENT
&SOLUTIONS

Act like someone’s life depends on what we do.

DISTRIBUTION STATEMENT A. Approved for Public Release.
• Modern engineering studies often rely on the use of computational models and simulations (M&S) to represent real-world system behavior
  
  – Broad applicability in engineering, as well as other research domains
  
  – M&S is also often referred to as ‘Model-Based Engineering’ (MBE), Model-Based Systems Engineering (MBSE), or ‘Computer-Aided Engineering’ (CAE)

• M&S involves the use of computers to generate data relating to the performance or behavior of a product, process, or system relative to various inputs using math models of the relationship between these variables.
  
  – This physics-based model may be very simple (closed-form ‘first-principle’ math model, such as: Kinetic Energy = \( \frac{1}{2} \text{Mass} \times \text{Velocity}^2 \)), or a highly complex, multiscale models, with open-form equations requiring high-performance computing systems to solve. In some instances a single iteration of the simulation may take hours, or even several weeks to converge.

• Regardless of model complexity, there are always simplifying assumptions made for the purpose of tractability in engineering M&S.
  
  – These assumptions often result in unaccounted-for uncertainty between the computational model predictions and ‘real-world’ system behavior. Verification & Validation (V&V) are key!
“The process of identifying all relevant sources of uncertainties, characterizing them in all models, experiments, comparisons of M&S results and experiments, and of quantifying uncertainties in all relevant inputs and outputs of the simulation or experiment.”

NASA Technical Standard 7009

- DOE provides the foundation
- Statistical, probabilistic, and numerical optimization techniques, model calibration, statistically rigorous V&V, and specialized Design of Experiments methods integrated with M&S
### OVERARCHING UQ FRAMEWORK

#### Computational Modeling & Simulation

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#### Planning

- **‘Live’ Testing & DOE-based Empirical Modeling**
- **Computational M&S Emulation / Reduced-Order Modeling**
- **Data Assimilation / V&V (M&S and Empirical)**

#### Results:

- Reliable, Robust, Optimized Products & Systems
- Credible and Realistic Engineering M&S Analytics
- Reduced Design Cycle Iterations; time to field
- ID’d opportunities to reduce manufacturing costs
- Reduced performance variation

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• Consider a simple hypothetical example:
• A complex analytical model is run (say, 12 hours per run), which produces virtual ‘data’ (much the same way live testing produces measurable data)
  – Deterministic – multiple runs w no change to inputs results in no change to output
  – Real life is not deterministic – variation is always present
• We cannot make realistic or credible statements about reality from M&S without accounting for variability and uncertainties!

Monte-Carlo methods can be used to represent natural variation in deterministic computational models

‘Brute force’ approach: 10,000 simulations would take around 14 years using the original model

Using the model emulator 10,000,000,000,000 simulations will now take a fraction of a second

For high-risk, rare-event simulations (< 1 / million safety critical) 10,000 simulations are clearly insufficient
UQ will reduce the computational demand for SDZ Evaluations by orders of magnitude by eliminating the need for direct execution of complex model:

<table>
<thead>
<tr>
<th>Cost</th>
<th>Schedule</th>
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<tr>
<td>Savings of &gt;10k labor per SDZ evaluation, plus other associated costs, vs. close to zero cost per SDZ evaluation once algorithm is developed</td>
<td>Each SDZ evaluation will now take fractions of a second instead of weeks</td>
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<td>→ Millions saved over several years</td>
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<th>Quality</th>
<th>Risk</th>
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<tr>
<td>Credible high-reliance models, with more customizable tradespace and sensitivity analyses</td>
<td>Significant improvement in precision of simulations for ‘one-in-a-million’ safety region</td>
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<td></td>
<td>→ From $10^4$ in weeks to $10^{15}$ in fraction of a second!</td>
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• Model calibration, Verification & Validation is used to determine the extent to which computational M&S represents true system behavior
• Various goals for assimilating computational data with ‘live’ testing
• ARDEC is beginning to leverage modern statistically rigorous V&V best-practices in our M&S evaluations
  – Credible, defensible models supported by testing
  – Represents technical risk to success of our supported programs, as well as end user safety risk
  – Prevent missed opportunities to shorten product design cycle, ID key parameters, and make systems more robust, resilient, and reliable
  – We need to better understand design margins, sensitivity to parameters, and suitability of our models!
• UQ is a rapidly growing field with broad applications
  – Recent explosion in powerful, cutting-edge V&V approaches and UQ methods
  – Solutions for DoD’s most difficult engineering problems!
Objective:
- To determine the variables with the strongest influence on the magnitude of the muzzle-exit axial and transverse accelerations observed in test data, and characterize their distributions.

Results:
- Proof-of-concept illustrates UQ approach and its ability to minimize actual M&S execution runs vs MC simulation approach
- Precise mapping of the entire 53 dimensional domain
- Statistically credible and defensible V&V of modeling & simulation with test data

UQ Approach:
- Monte Carlo-based approach used with FORTRAN model as initial proof of concept with 13,000+ sims
- Alternatively, Hybrid DOE approach (Fraction Factorial & Optimal Space-Filling) using ‘SmartUQ’ – accurate model with less than 100 sims
- Variance decomposition and sensitivity analysis methods used to rank-order parameters by impact
- Inverse Uncertainty Analysis used to develop probability distributions for top 10 parameters resulting in ‘live’ test data variation observed in 16 firings
Sequential Testing is a long-standing DOE best-practice

Inform each stage of testing from previous test results making for more efficient use of resources

This approach can be followed for ‘physical’ DOE’s (‘live’ testing) as well as DOE for M&S / UQ
SEQUENTIAL TEST STRATEGY: 
SUBSYSTEM TO SYSTEM-LEVEL

X's

Subsystem 1

Plan 
Execute
Design 
Analyze
Screening Experiment 
Characterization Experiment 
Response Surface Experiment

Subsystem 2

Screening Experiment 
Characterization Experiment 
Response Surface Experiment

Subsystem n

Y's

System-level Experimentation / UQ

X's

Screening Experiment 
Characterization Experiment 
Response Surface Experiment

Y's
CASE STUDY
• **Objective**: determine which individual components of a fire control system are worth investing in improvements by determining which have the largest effect on the probability of hit of a weapon system

• Fire Control System

• Ballistic Trajectory Model

• Analysis Methodology
  • Experimental Design
  • Surrogate Modeling
  • Sensitivity Analysis

• Conclusions
• **Purpose:**
  • Determine the correct position to aim a weapon in order to hit a target

• **Function:**
  • Collects input data
    • Environmental conditions such as temperature, pressure, humidity and wind
    • Target location in range and elevation
  • Calculates the necessary placement of the gun barrel in terms of elevation and azimuth angles required to hit the target
  • Positions the weapon based on those angles
• A modified 3-DOF model that solves the equations of motion for a bullet from the time it exists the gun until it impacts its target
  • Dependent on velocity, air density, drag, wind, etc.

• For this effort, model used to predict miss distance of a shot given known errors in the input conditions

• Example: Shooter thinks there is a crosswind of 5 mph, but the crosswind is actually 7 mph
  • Shooter thinks the crosswind is 5 mph, so the model calculates where he needs to aim the gun to hit a target in 5mph winds
  • Assuming he fires using that calculated aim point, the model then calculates the bullet trajectory in 7 mph wind.
  • Result is the difference between the target location and impact location
• Design of Experiments (DOE): Best practice test method to generate valid, useful data to support the development of an accurate model
  • Input variables (factors) are systematically varied to identify the effect that each variable has on the outputs of interest
  • Optimize performance, reliability, and safety
• Design and Analysis of Computer Experimentation (DACE)
  • DOE for deterministic computational experimentation
  • Addresses real-world uncertainty not present in deterministic simulation through uncertainty quantification (UQ)

• Design the experiment
  • Factor, responses
  • Space filling design
• Fit a surrogate model
• Utilize the surrogate model to achieve the desired objective
### Experimental Design

**Environmental Factors**

- Pressure
- Temperature
- Humidity
- Cross Wind
- Range Wind
- Latitude
- Bearing

**Sensor Error Factors**

- Pressure Error
- Temperature Error
- Humidity Error
- Cross Wind Error
- Range Wind Error
- Latitude Error
- Bearing Error

**Responses**

- $\Delta Z$
- $\Delta Y$

- Factors are the individual sensors of the fire control system that we are interested in
- Design and Noise Factors present
- Responses are the miss distances

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**Space Filling Design:** Systematically determines which points in the design space are sampled within the ballistic trajectory model

- 400 points uniformly placed throughout the design space
- Run the ballistic trajectory models, fit the surrogate models
Parameter Importance via Variance Decomposition

- Surrogate model fit to both miss distance ($\Delta Z, \Delta Y$) outputs

- Global sensitivity analysis of each factor to the response obtained
  - Cross wind error accounts for 84% of the variation in the horizontal ($\Delta Z$) direction
  - Range error accounts for 87% of variation in the vertical ($\Delta Y$) direction
Prediction Profiler

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Interactions

• Profiler only shows how each factor changes the response when all other factors remain constant.

• All the environmental factors were flat because when all the errors equal 0, the simulated miss distance will always be 0.

• This does not mean that there is no effect due to these factors when errors are not 0.

• Interaction between pressure and crosswind error for the horizontal miss distance and pressure and range error for the vertical miss distance.

\[ \Delta Z \]
\[ \Delta Y \]

Cross Wind Error

Range Error
Baseline Design

Defect Rate

\[
\begin{align*}
\Delta Z & \quad 0.61725 \\
\Delta Y & \quad 0.25875 \\
\text{All} & \quad 0.69998
\end{align*}
\]
Improved Designs

• Iteration 0: P(hit) = 30%

• Iteration 1: significantly improve crosswind and range sensors: P(hit) = 78%

• Iteration 2: moderately improve pressure and temperature sensors: P(hit) = 86%
CONCLUSION

- Designed an experiment using a ballistic trajectory model to determine the miss distance between a bullet and its target given certain errors in the fire control system.

- Fit a model to the results that was able to assess the overall importance of each sensor (cross wind, range) and determine in which conditions the sensors were most important in (high pressure, low temperature).

- Executed probabilistic MC simulation-based robustness study through error propagation to determine $P[\text{Hit}]$ for given precisions of individual sensors.

- Credible results to optimize this small-arms fire control design.