



INSTITUTE FOR DEFENSE ANALYSES

## **Implementing Fast Flexible Space-Filling Designs in R**

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## Executive Summary

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Modeling and simulation (M&S) is a critical component of testing and evaluating major weapon systems in the Department of Defense. When planning M&S, testers use experimental design techniques to determine how much and which types of data to collect. When running M&S, analysts can explore multiple experimental design methodologies to apply when determining the best conditions to test.

Sometimes, testers employ full-factorial designs when planning a test using M&S. However, when designing a test that involves M&S, testers can use space-filling designs (SFDs) to better spread out points across the operational space. Fast flexible space-filling designs (FFSFDs) are a type of SFD useful for M&S because they work well in design spaces with disallowed combinations (i.e., nonrectangular design spaces) and permit the inclusion of categorical factors. Both nonrectangular design spaces and categorical factors are recurring features in defense testing.

An example of the use of an FFSFD is the planning of M&S for missile testing.

Guidance from the Deputy Secretary of Defense and DOT&E encourages the use of open and interoperable software and recommends the use of SFDs.<sup>1</sup> This project addresses the directives of these two memoranda.

IDA analysts developed a function to create FFSFDs using the free statistical software R. To our knowledge, no R packages for creating an FFSFD can accommodate a variety of users' inputs, such as categorical factors. Moreover, users of IDA's function can share their code to make their work reproducible. IDA plans to make the function available through the Comprehensive R Archive Network (CRAN), which hosts other contributed R packages.

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<sup>1</sup> See Deputy Secretary of Defense memorandum, May 2021, "Creating Data Advantage"; and DOT&E memorandum, January 2017, "Clarifications on Guidance on the Validation of Models and Simulation Used in Operational Test and Live Fire Assessments."



# Implementing Fast Flexible Space-Filling Designs in R

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JSM 2023

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## Bottom Line Up Front

**Motivation:** Space-filling designs are desirable for planning modeling and simulation test campaigns

**Problem:** No R function that provides space-filling designs

**Project :** Developed a function in R that generates fast flexible space-filling designs and can:

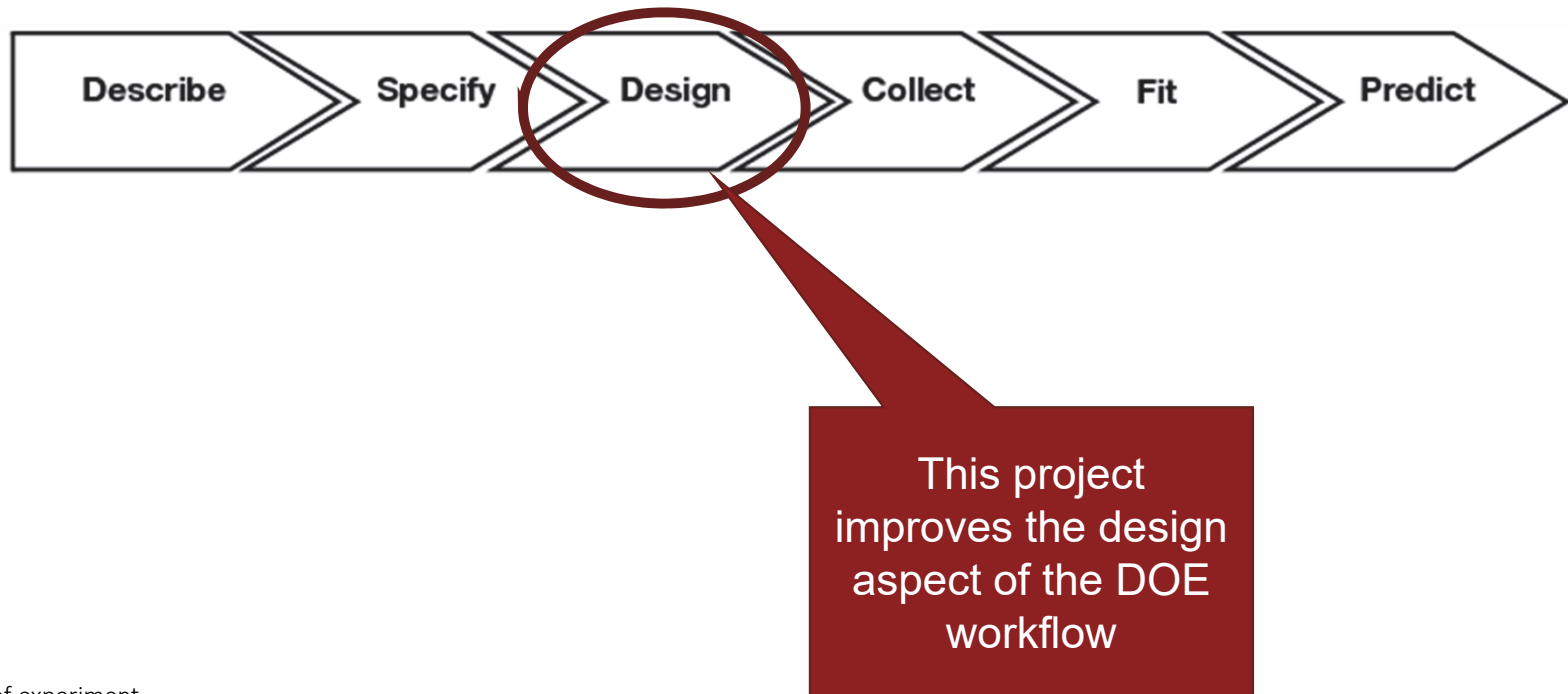
- Handle continuous and categorical factors
- Accommodate user inputs

# IDA leverages DOE techniques to choose test conditions when planning a test for a military system



DOE = design of experiment

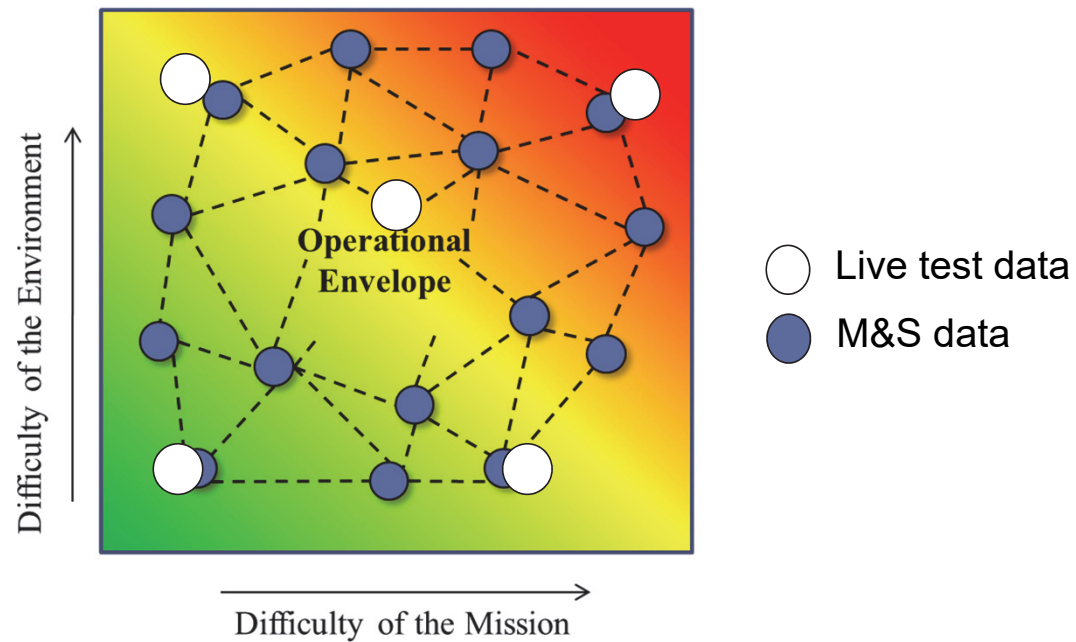
# IDA leverages DOE techniques to choose test conditions when planning a test for a military system



DOE = design of experiment



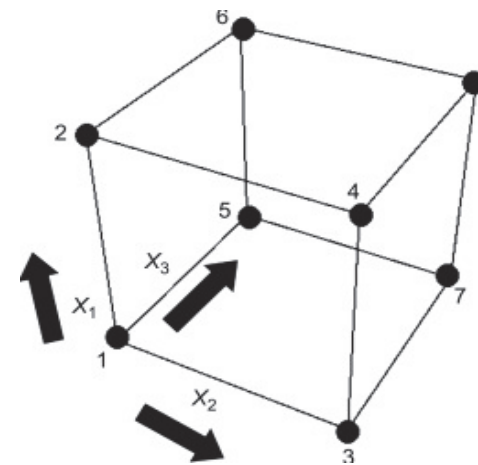
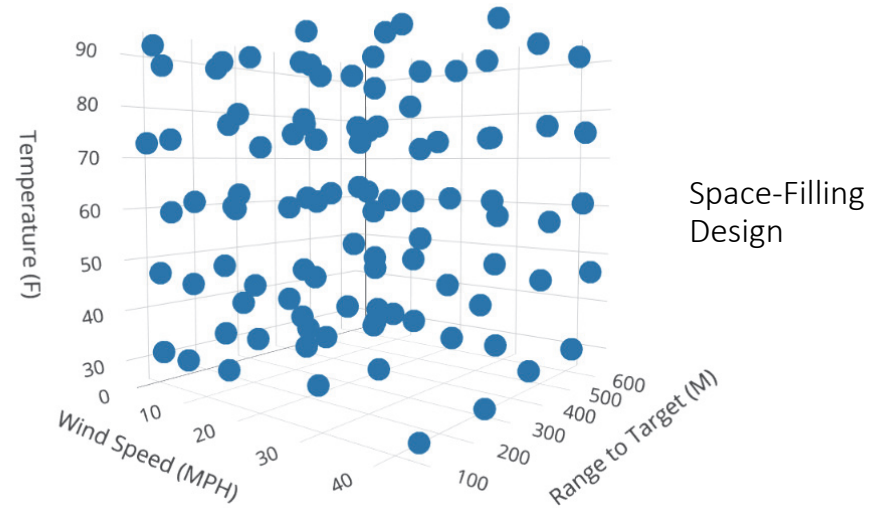
# M&S can be used to supplement real-life test data



M&S = modeling and simulation

# Overview of Space-Filling Designs

- Space-filling designs are used to spread out points across the operational space
- Space-filling designs differ from classical designs, which tend to push points to the exterior of the operational space



M&S = modeling and simulation

## Our R program creates an FFSFD, a type of space-filling design

- Operational tests often require continuous and categorical variables
- An FFSFD can take categorical and continuous data as inputs and can accommodate nonrectangular design spaces
- Most other space-filling designs apply only to continuous factors

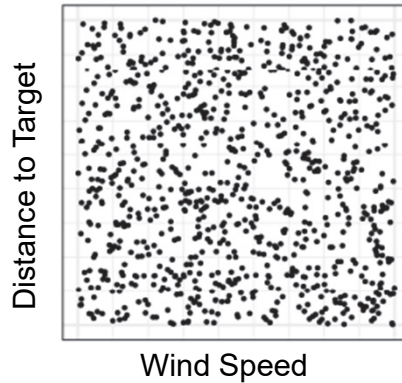
Categorical Data Examples	Continuous Data Examples
Rainy/sunny/cloudy	Temperature
Windy/not windy	Wind speed
Launch platform of a missile	Distance to target

FFSFD = fast flexible space-filling design

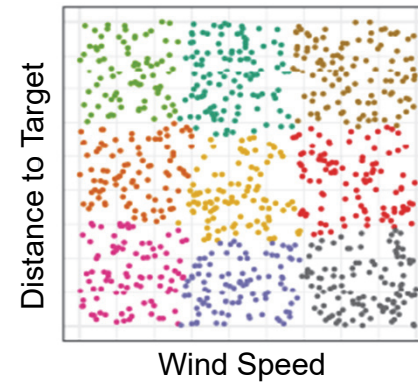
\* The R program implements the algorithm Ryan Lekivetz and Bradley Jones propose in their 2019 paper, "Fast Flexible Space-Filling Designs with Nominal Factors for Nonrectangular Regions."

# The FFSFD algorithm ensures that design points cover the entire design space (continuous data example)

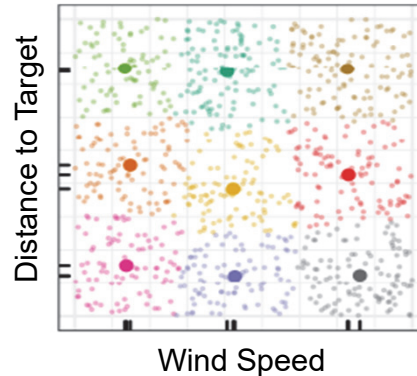
**Step 1:** Generate random points in design space



**Step 2:** Group points into primary clusters



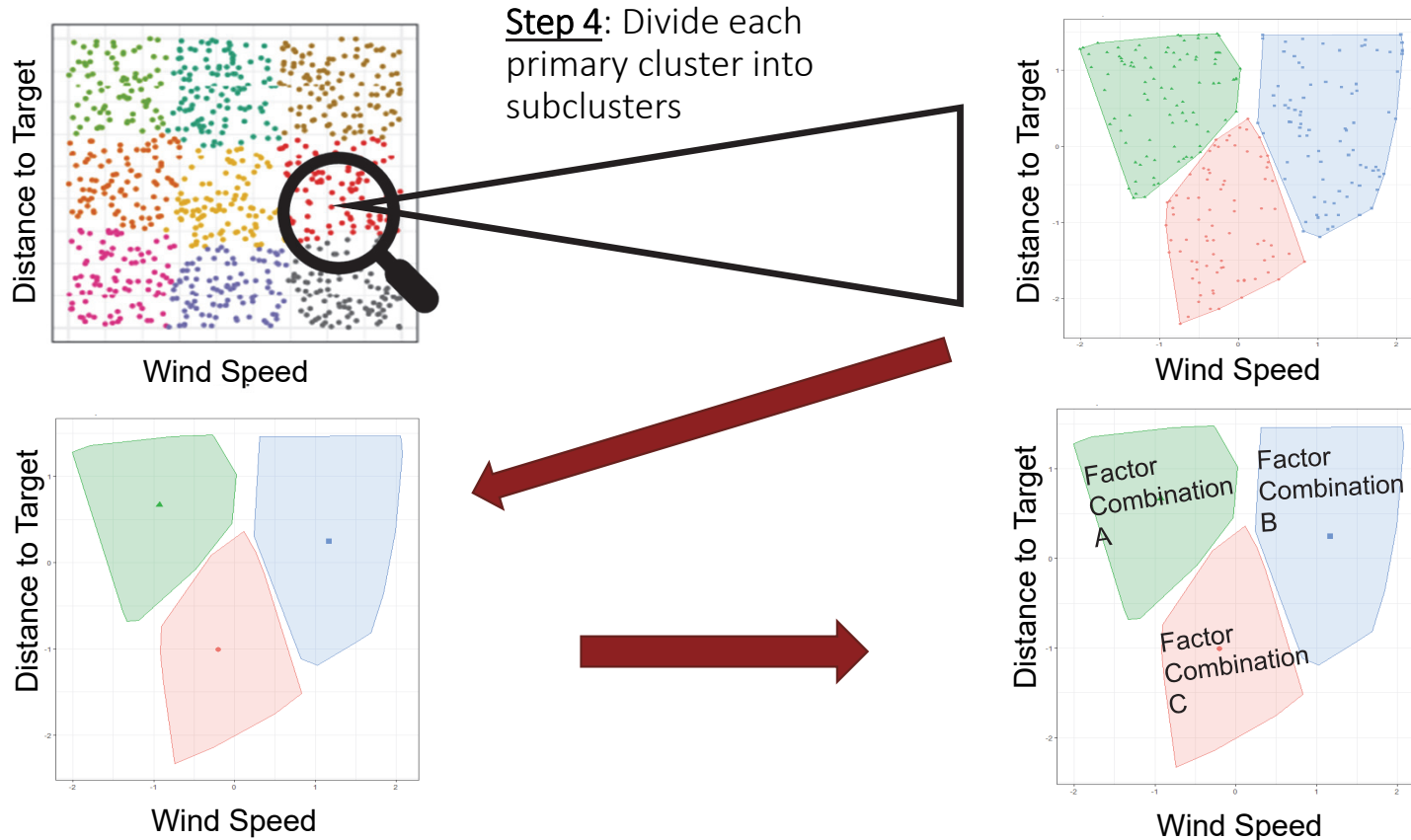
**Step 3:** Obtain center point of each cluster



FFSFD = fast flexible space-filling design

Heather Wojton et al., 2021, *Space Filling Designs for Modeling and Simulation Validation*, IDA Document NS D-21562 (Alexandria, VA: Institute for Defense Analyses)

# For categorical variables, FFSFD generates secondary clusters within each primary cluster

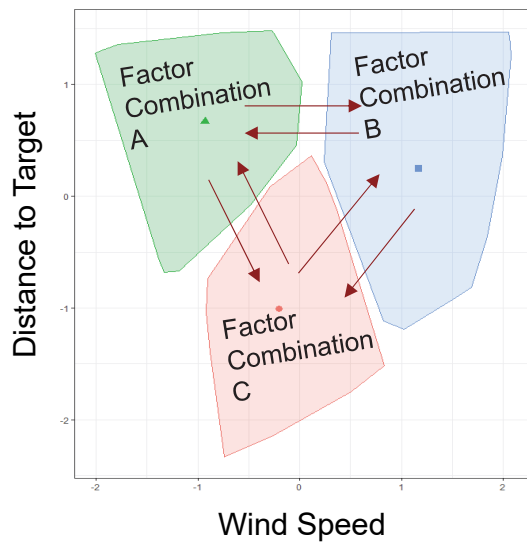


FFSFD = fast flexible space-filling design

Heather Wojton et al., 2021, *Space Filling Designs for Modeling and Simulation Validation*, IDA Document NS D-21562 (Alexandria, VA: Institute for Defense Analyses)

Ryan Lekivetz and Bradley Jones, 2014, "Fast Flexible Space-Filling Designs for Nonrectangular Regions," *Quality and Reliability Engineering International* 31, no. 5, 829–837

## Use an optimization criterion to choose the final design



Step 7: Randomly switch the labels, and we choose the design based on an optimality criterion



Icon Source: Optimize by Markus from NounProject.com

Joseph, V. Roshan, et al. "Maximum Projection Designs for Computer Experiments." *Biometrika*, vol. 102, no. 2, 2015, pp. 371–380., <https://doi.org/10.1093/biomet/asv002>.

# Implementation of FFSFD: Missile System Example

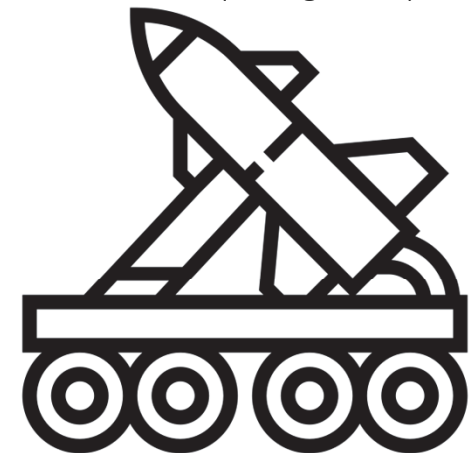
Wind Speed  
(Continuous)



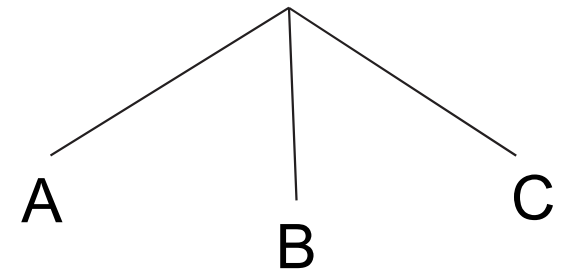
Distance to Target  
(Continuous)



Launch Platform  
(Categorical)



**Response:** Missile Accuracy (Miss Distance)



FFSFD = fast flexible space-filling design

Icon source: NounProject.com

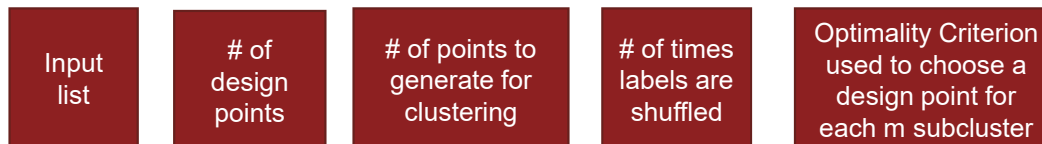
## This R package is free, open-source software

```
d <- list(wind_speed = c(1, 500),  
         range_to_target = c(1, 800),  
         platform = c("A", "B", "C"))
```

Step 1: Generate list of inputs

```
fff(d, n, N, r = 10, c)
```

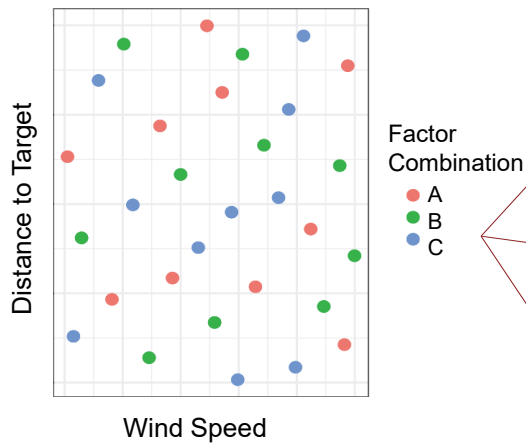
Step 2: Plug into fff function and generate design



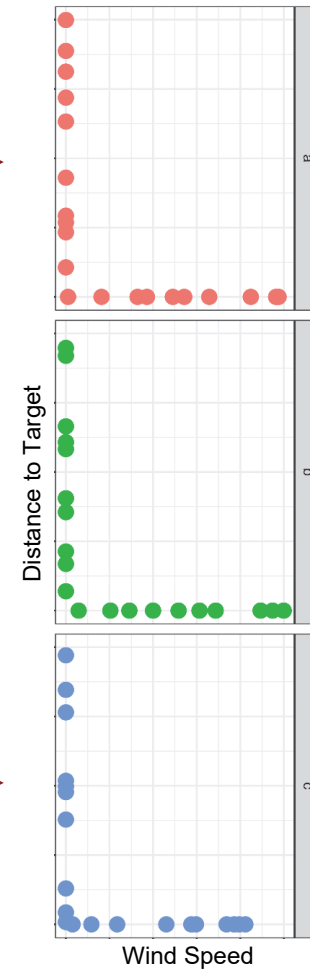
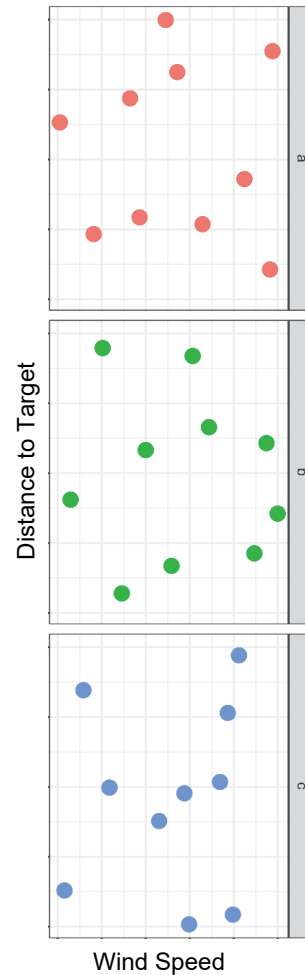


# The result is a space-filling design that takes into account categorical factors

Overall SFD with 3 factors



Mini SFD in each factor combination

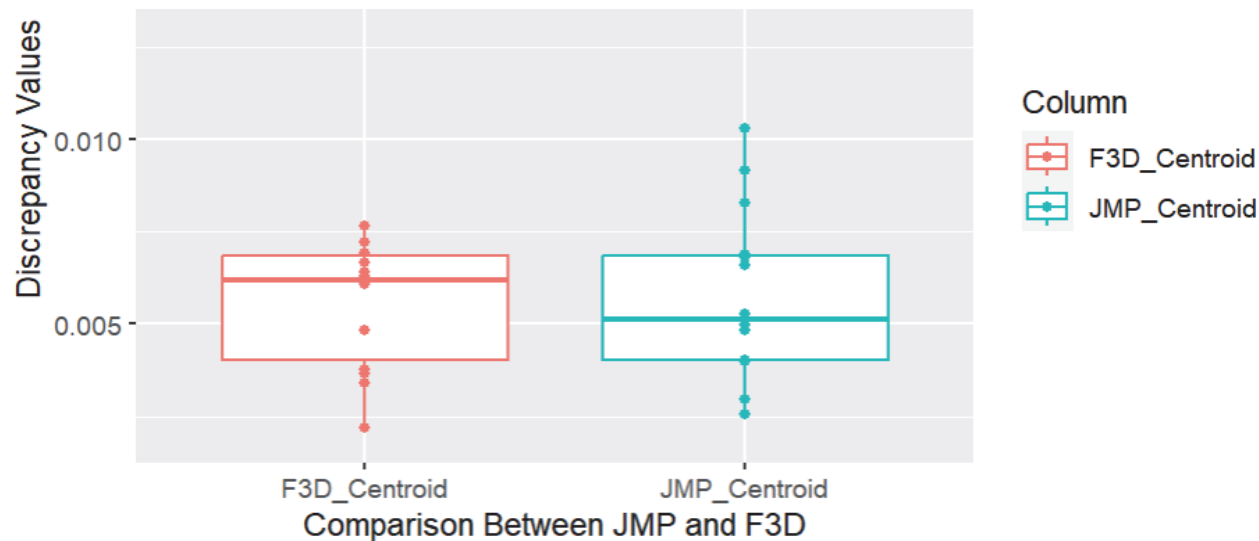


The SFD has good projection properties

SFD = space-filling design

## Comparison of R package discrepancy measures with JMP

- To validate the accuracy and reliability of our designs with JMP, we employed a paired t-test to compare the mean discrepancy measures between our software methods and JMP to determine whether they were statistically significant.
- Discrepancy measures assess the extent to which the number of sample points included in subspaces of a design are close to the subspaces' volume.



## Closing Remarks

- Fast flexible space-filling designs can be used when working simultaneously with continuous and categorical data
- Our R package generates fast flexible space-filling designs and can be used for future modeling and simulation work
- **Coming soon:** Function will be published as an R package on CRAN and [testscience.org](https://testscience.org)

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