



INSTITUTE FOR DEFENSE ANALYSES

**Space-Filling Experimental Design and Surrogate Models  
for U.S. Department of Defense Modeling and Simulation  
Evaluation**

John T. Haman, Project Leader

Curtis G. Miller

OED Draft

August 2023

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IDA Document 1038122

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#### About This Publication

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**Space-Filling Experimental Design and Surrogate Models for U.S. Department of  
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## Executive Summary

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Operational testing (OT) provides important information on the capability of equipment acquired by the United States Department of Defense (DoD). Operational testing has been a vital component of the acquisition process since Congress reformed OT conduct with the Department of Defense Authorization Act of 1984. Computer modeling and simulation (M&S) can help address the small sample size problem in live, operational testing, but M&S must be validated in order for stakeholders to believe M&S makes meaningful predictions about real-world outcomes.

The Institute for Defense Analyses (IDA) has published documents providing guidance to the OT community on statistical validation of M&S, including a handbook and papers on the use of space-filling designs and statistical surrogates (also known as metamodels). However, important questions remain that need to be answered in order to best validate M&S.

In particular, operational testers need better recommendations on sample size selection, determining the number of replicates in a design (if any), validating with small real-world sample sizes, and incorporating statistical

surrogates into hypothesis tests that help determine whether M&S outcomes match real-world outcomes or not.

We hope the larger statistical community can help contribute answers to these questions. This presentation was given at the Joint Statistical Meeting 2023 in Toronto, Canada.



# Space-Filling Experimental Design and Surrogate Models for U.S. Department of Defense Modeling and Simulation Evaluation

Dr. Curtis Miller

August 9, 2023

**Institute for Defense Analyses**

730 East Glebe Road • Alexandria, Virginia 22305

# We want statisticians to appreciate...



*... why operational testing matters*



*... why computer modeling and simulation matters to operational testing*



*... the challenges in planning computer modeling and simulation studies*



*... the challenges in statistically validating a computer model*

Why does operational testing matter?



# Operational testing studies DOD system effectiveness in warfighting conditions

97 STAT. 614 PUBLIC LAW 98-94—SEPT. 24, 1983

Public Law 98-94  
98th Congress

An Act

Sept. 24, 1983  
[S. 675]


To authorize appropriations for fiscal year 1984 for military operations, for research, development, test, and evaluation, for procurement, and for maintenance, to prescribe personnel strengths for the Armed Forces and for civilian employees of the Department of Defense, and for such other purposes as may be necessary in the interest of national defense.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled,

SHORT TITLE; TABLE OF CONTENTS

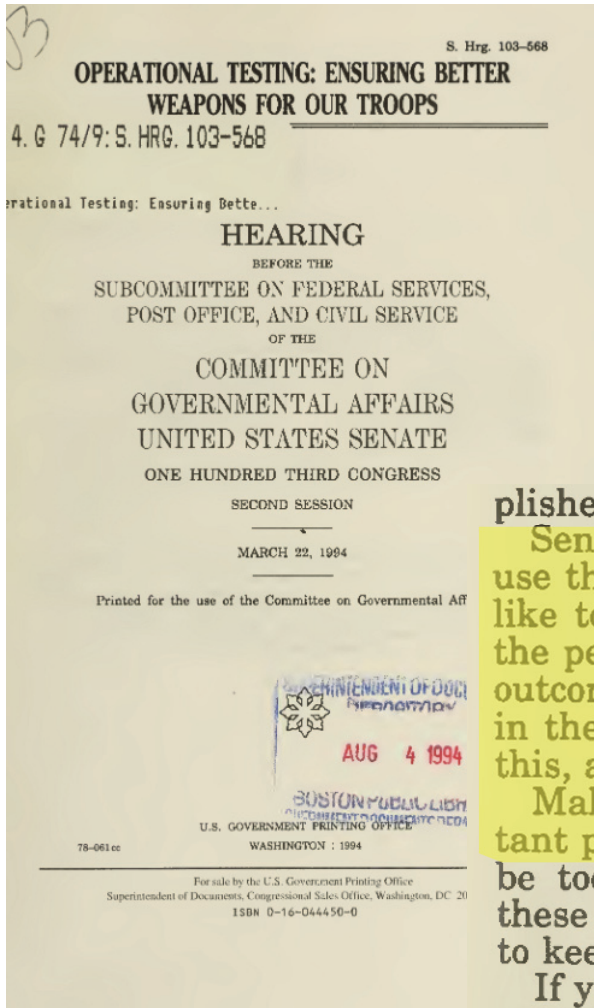
SECTION 1. (a) This Act may be cited as the "Department of Defense Authorization Act, 1984".  
(b) The table of contents for this Act is as follows:

Department of Defense Authorization Act, 1984.





# OT results matter to both Congress and the warfighter



Sen. D. Pryor (D-AK)



Sen. W. Roth (R-DE)

plished.

Senator PRYOR. Now, you know, testing is not a very—I hate to use the word “sexy,” but it is not a very high priority item it seems like today. There are not a lot of people interested in this, except the people in the battlefield. They are going to be interested in the outcome of all of this, and they are certainly going to be interested in the outcome of S. 1587. They will also be interested in whether this, as we call it, Mack truck amendment, remains in the bill.

Making certain that these weapons work is a very, very important part of our military preparedness. Whoever our enemies might be today, we had better believe that their intelligence knows if these weapons work or don't work, and I think that we have got to keep that in mind throughout this whole process.

If you would please continue.

Mrs. PRESTON. Let me quickly go through some of the other

# OT&E should reduce risk and uncertainty regarding the performance of systems in wartime

GAO

United States General Accounting Office  
Report to the Honorable  
William V. Roth and the Honorable  
Charles E. Grassley, U.S. Senate

October 1997

## TEST AND EVALUATION

### Impact of DOD's Office of the Director of Operational Test and Evaluation



number of unknowns prior to the decision to begin full production, while program and service officials typically sought less testing and were willing to accept greater risk when making production decisions. The additional testing DOT&E advocated, often over the objections of service testers, served to meet the underlying objectives of operational testing—to reduce the uncertainty and risk that systems entering full-rate production would not fulfill their requirements.

GAO/NSIAD-98-22

Why does computer modeling and simulation matter to OT?



# Small data problems are alive and well in operational testing

*SM-3 Blk IIA*



*Next Generation Interceptor*



*PAC-3*



***Missile tests can cost \$10Ms to \$100Ms  
per shot***

# Small data problems are alive and well in operational testing

*SM-3 Blk IIA*



*Next Generation Interceptor*



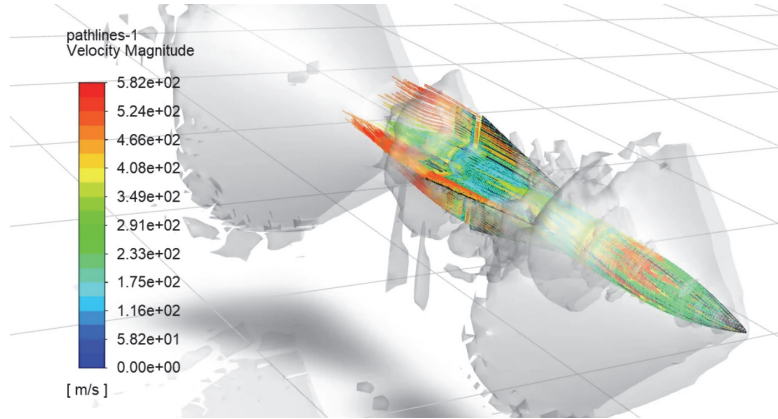
*PAC-3*



**COST AND ASSET  
AVAILABILITY GENERATE  
DEMAND FOR M&S**

***Missile tests can cost \$10Ms to \$100Ms  
per shot***

# Modeling and simulation comes in a variety of flavors with unique statistical considerations



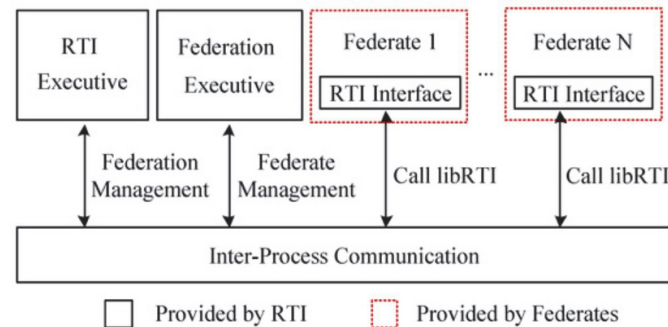
## Digital Simulation

[https://www.reddit.com/r/dcsworld/comments/mqmv3/lowquality\\_steadystate\\_cfd\\_simulation\\_of\\_an\\_aim54](https://www.reddit.com/r/dcsworld/comments/mqmv3/lowquality_steadystate_cfd_simulation_of_an_aim54)



## Hardware in the Loop

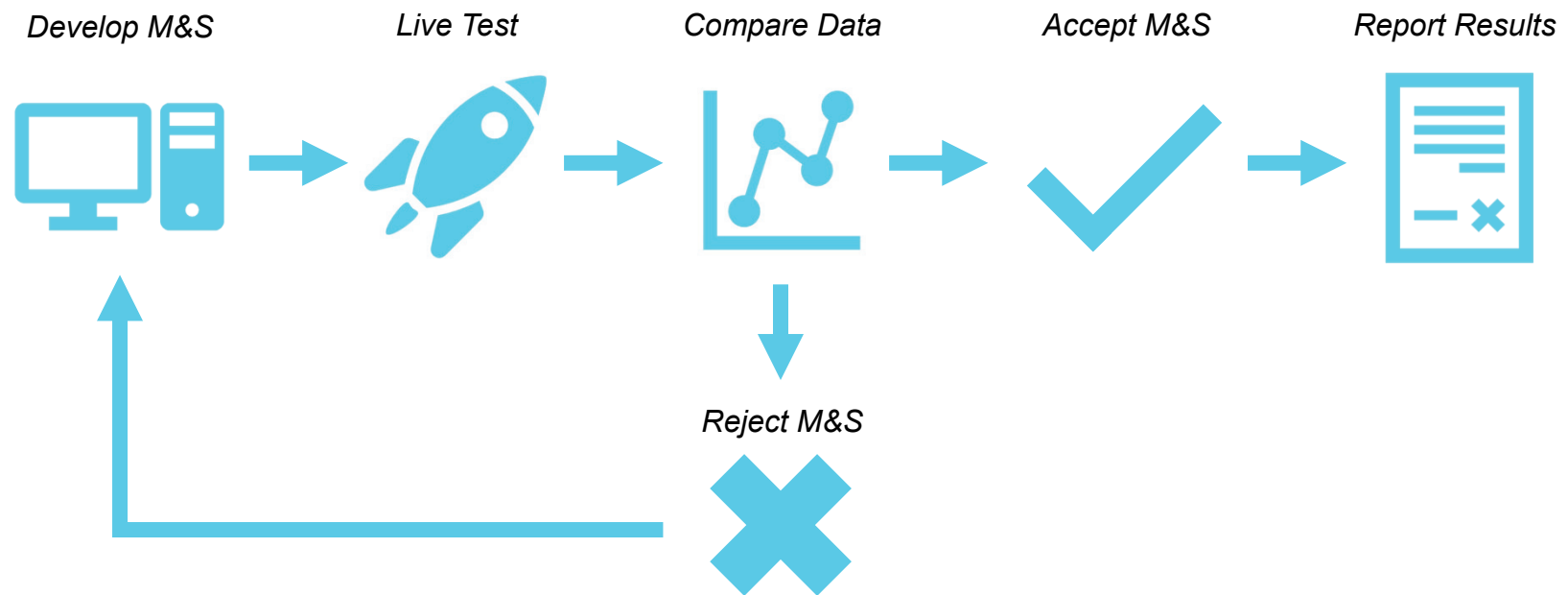
<https://testscience.org>



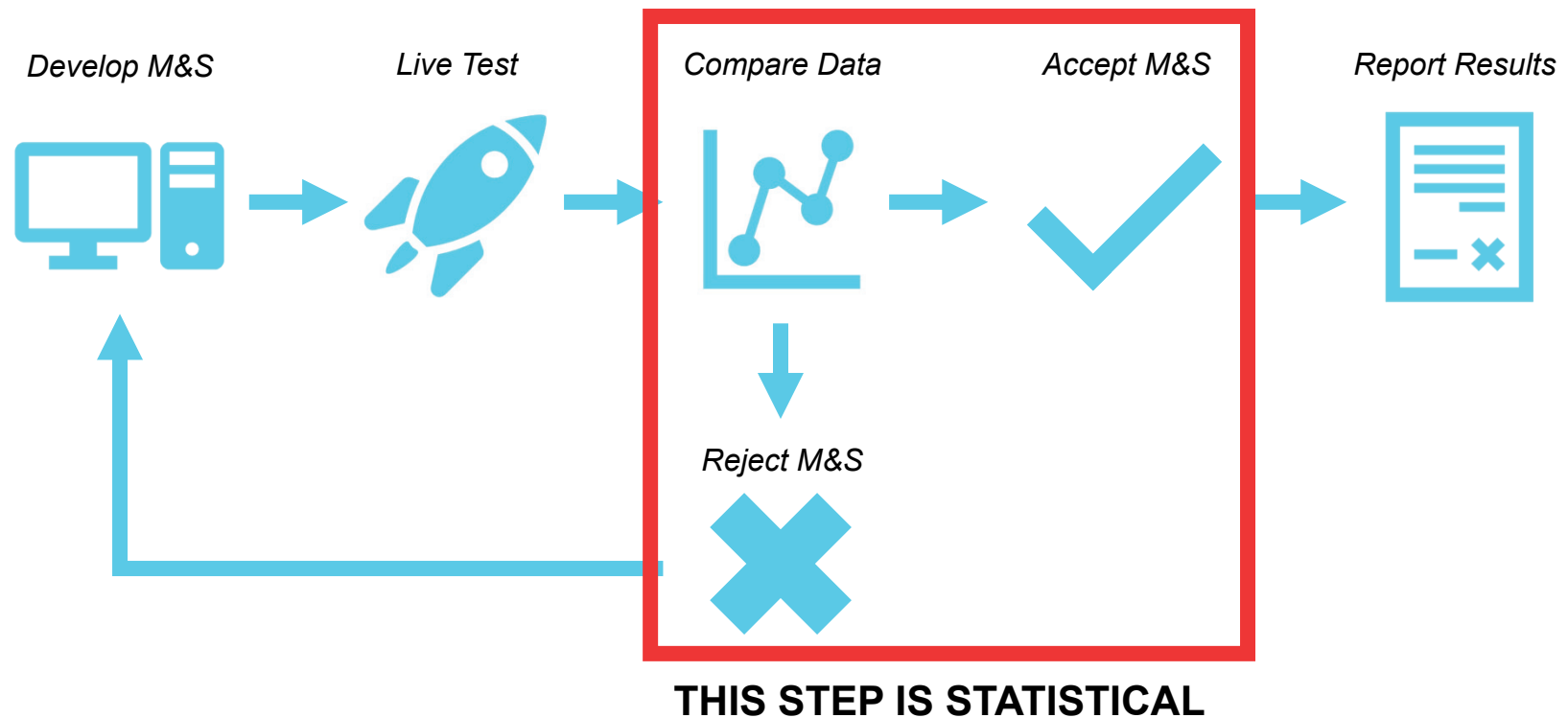
## Federation

<https://doi.org/10.3390/electronics9030540>

If M&S outputs will be used for predicting OT results, we must compare M&S outputs to live test data



If M&S outputs will be used for predicting OT results, we must compare M&S outputs to live test data





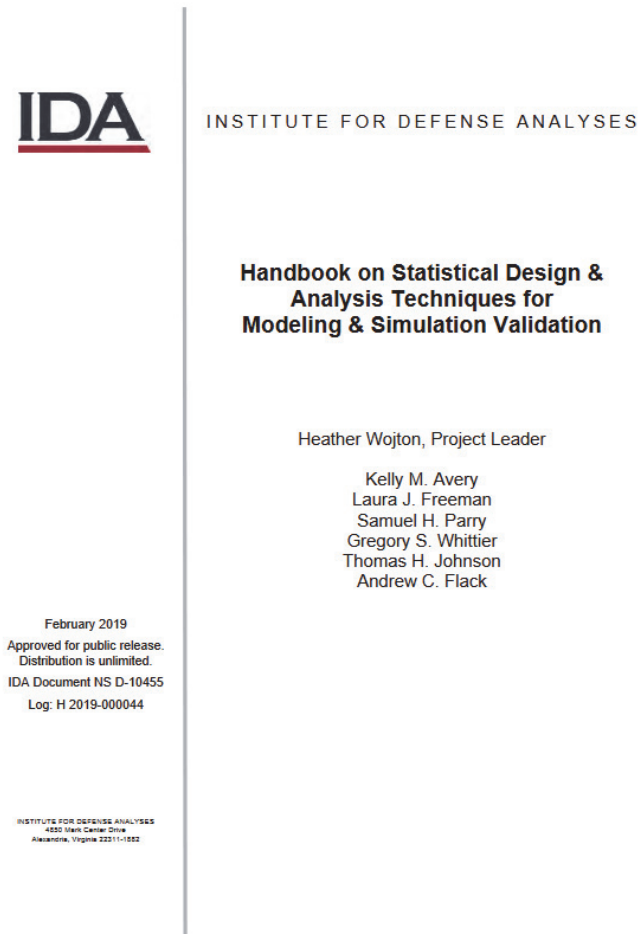
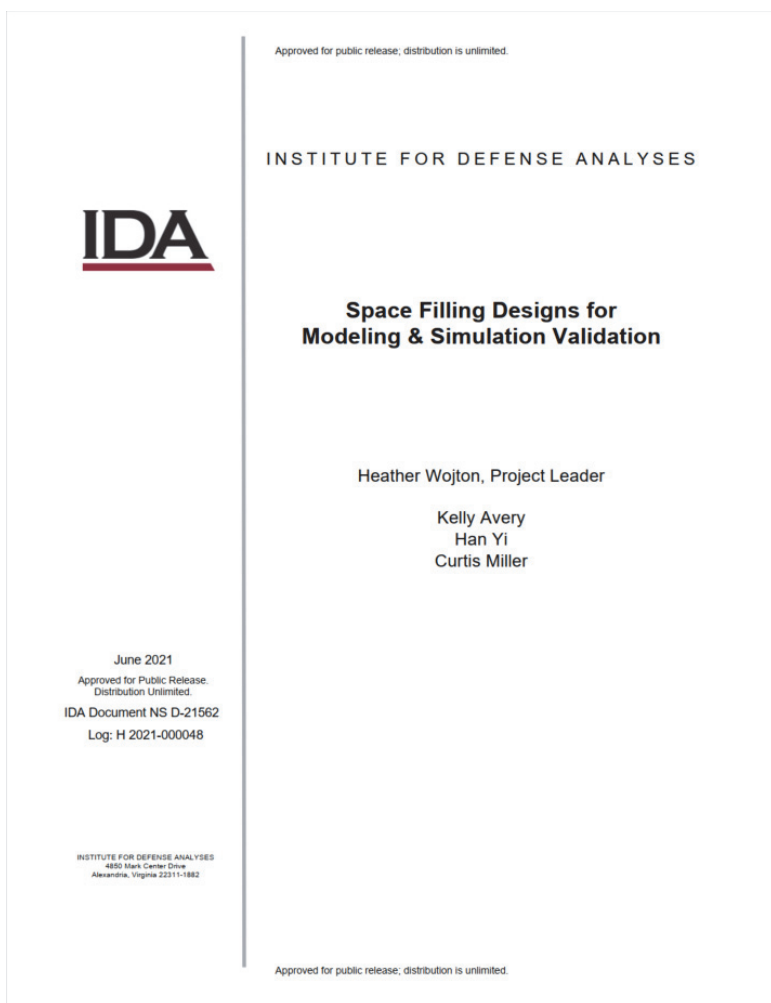
If M&S outputs will be used for predicting OT results, we must compare M&S outputs to live test data



What are the challenges in planning computer M&S studies?



# IDA publications introduce and recommend M&S DOE best practices



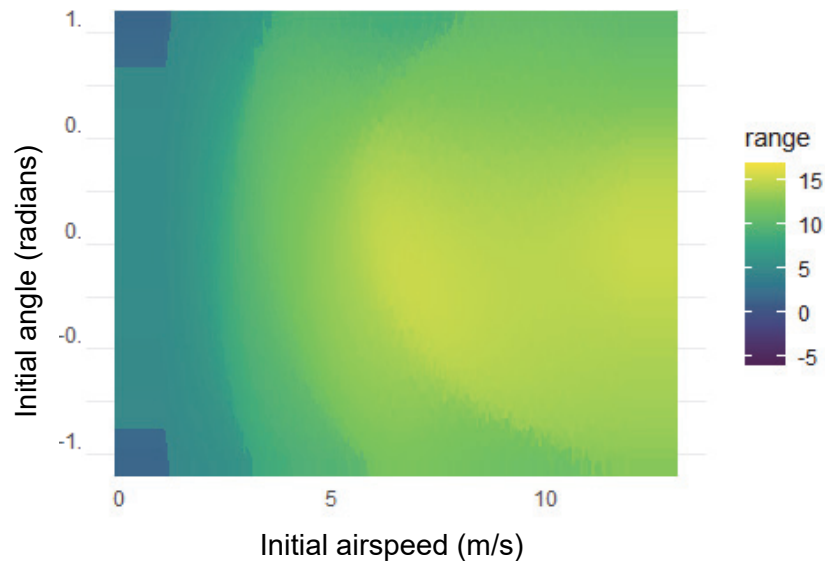
DOE – Design of experiments; IDA – Institute for Defense Analyses; SFD – Space-Filling Design

[https://testscience.org/wp-content/uploads/formidable/20/SFD\\_Literature\\_Review\\_Final.html](https://testscience.org/wp-content/uploads/formidable/20/SFD_Literature_Review_Final.html)

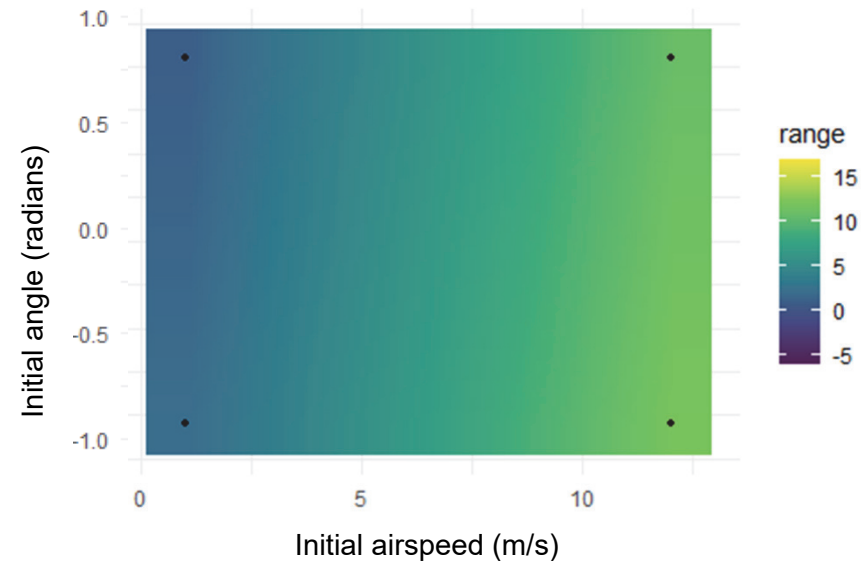
# Space-filling design of experiments helps recover more trends in simulation outputs

A **factorial** design with a simple **linear model** fitted will not accurately describe the M&S system's behavior.

**True response surface of simulated projectile range**

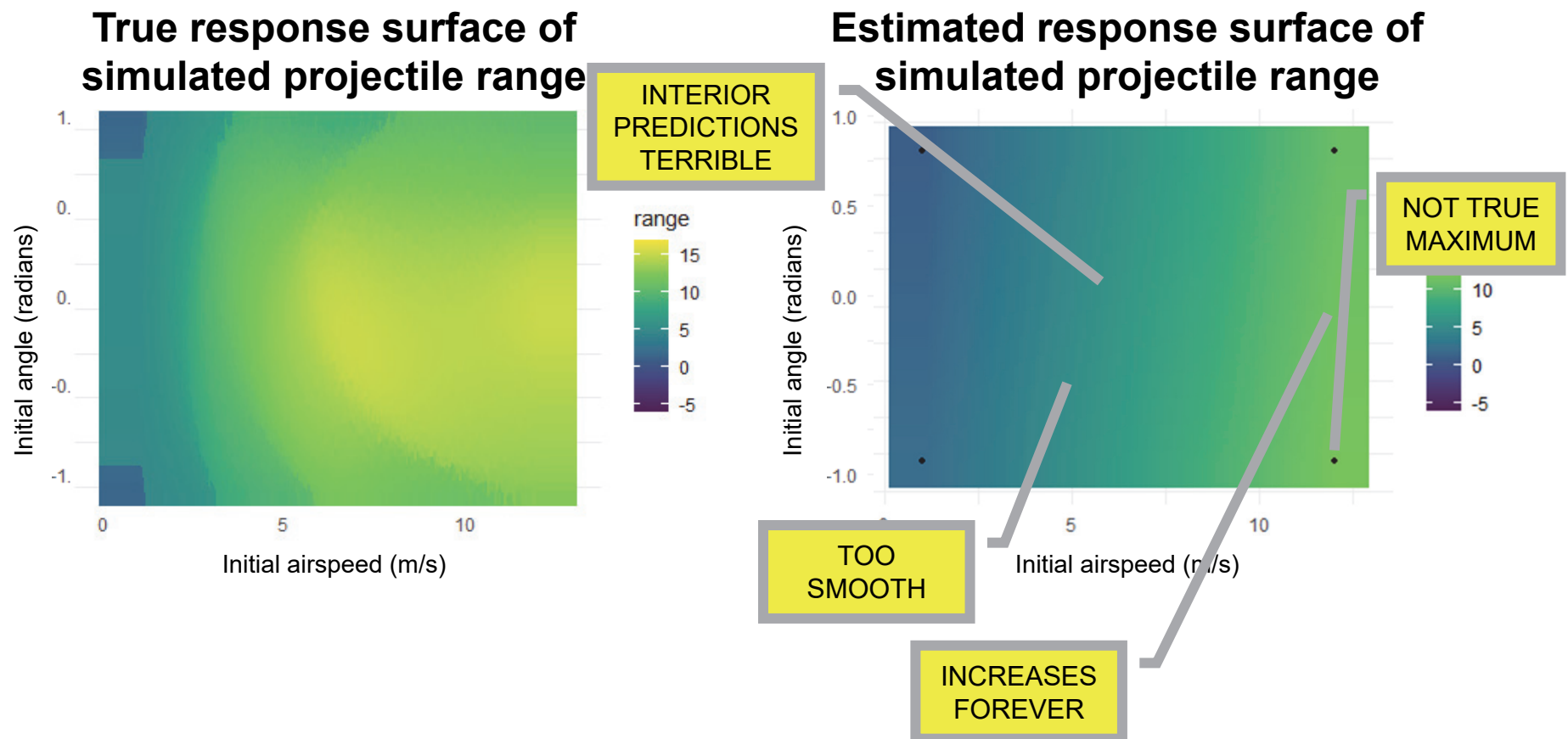


**Estimated response surface of simulated projectile range**



# Space-filling design of experiments helps recover more trends in simulation outputs

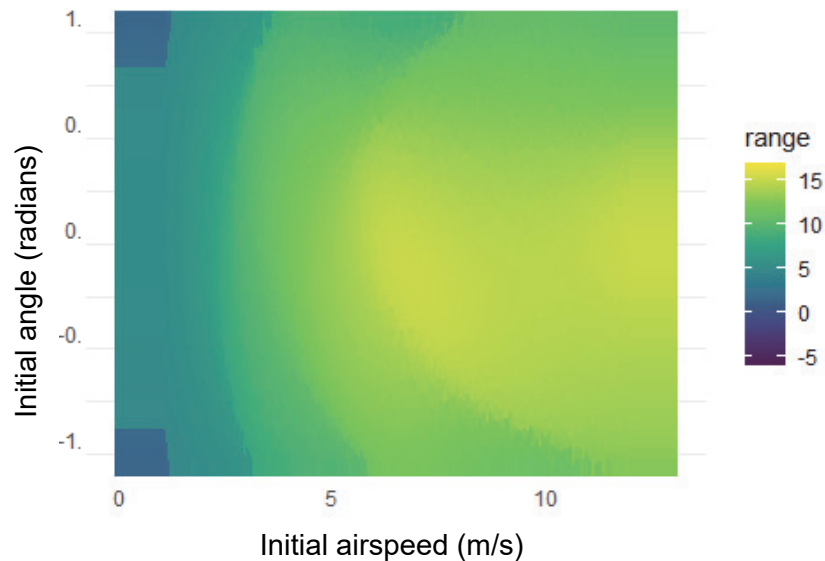
A **factorial** design with a simple **linear model** fitted will not accurately describe the M&S system's behavior.



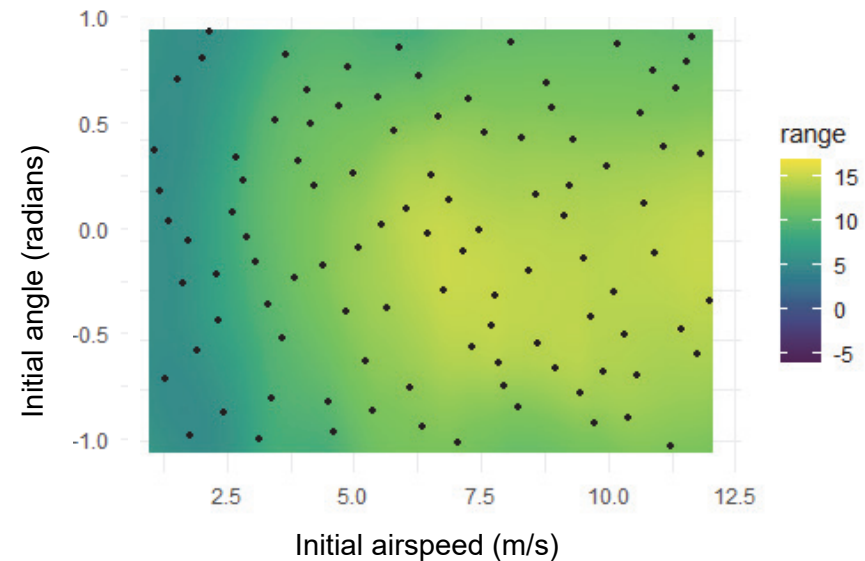
# Space-filling design of experiments helps recover more trends in simulation outputs

Analyzing the flights with a **Gaussian Process model** via a **Space-Filling Design** yields a good approximation to simulation output.

**True response surface of simulated projectile range**

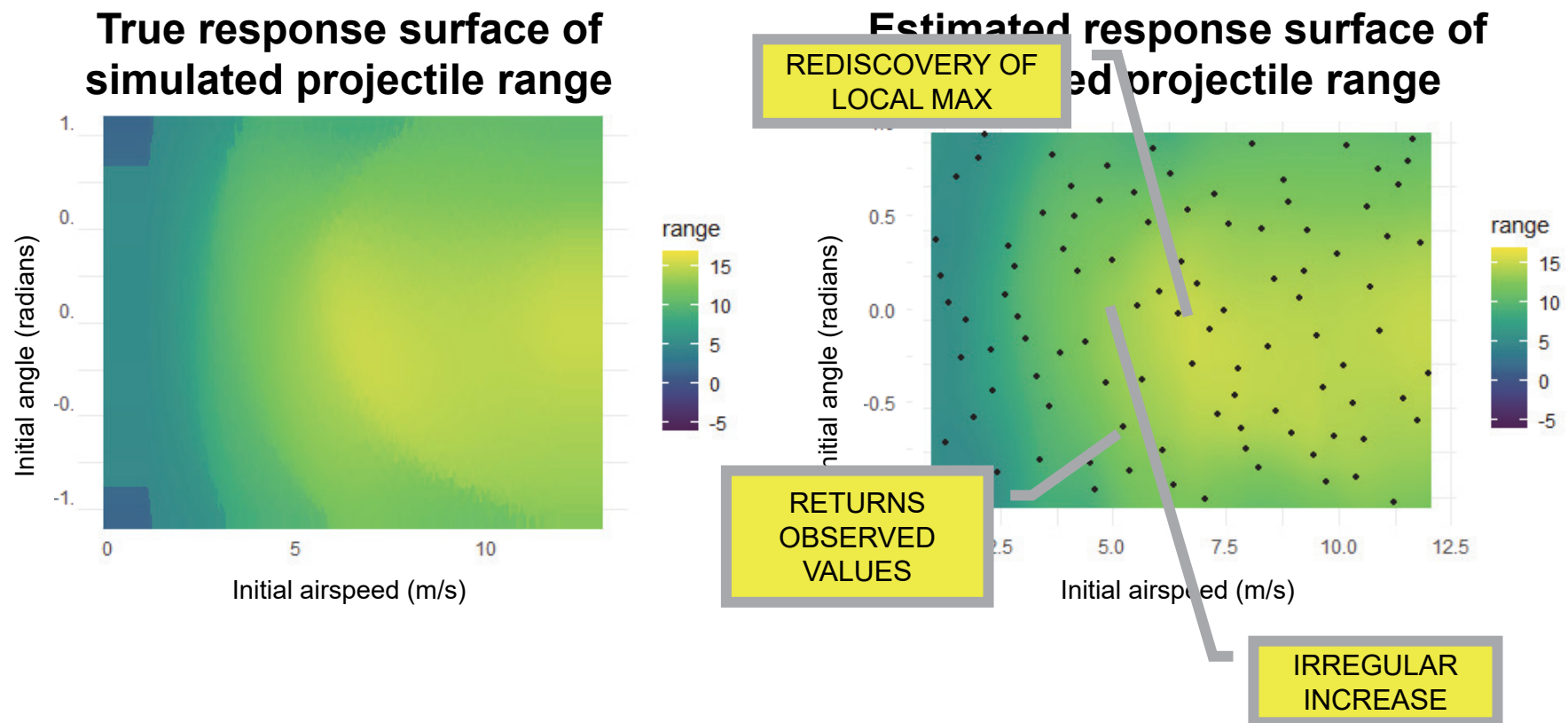


**Estimated response surface of simulated projectile range**



# Space-filling design of experiments helps recover more trends in simulation outputs

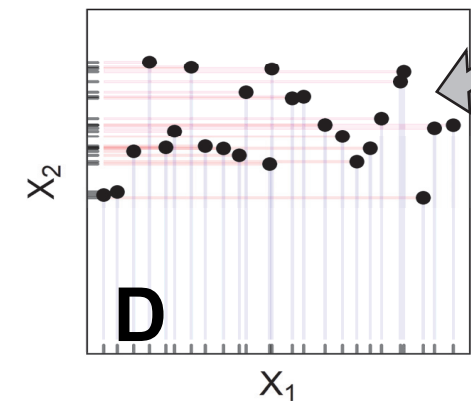
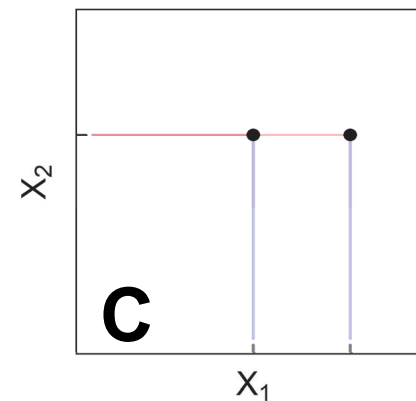
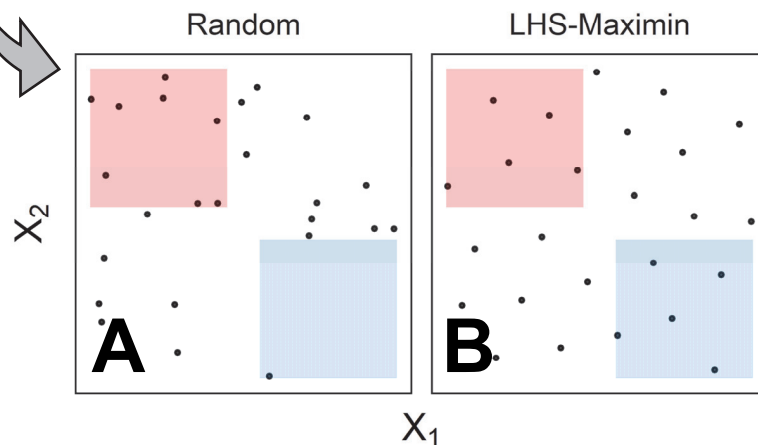
Analyzing the flights with a **Gaussian Process model** via a **Space-Filling Design** yields a good approximation to simulation output.



# Just like with classical DOE, there are quantitative ways to evaluate a specific design

Many criteria exist, but it is particularly important that an SFD satisfy the following three criteria in order to be useful:

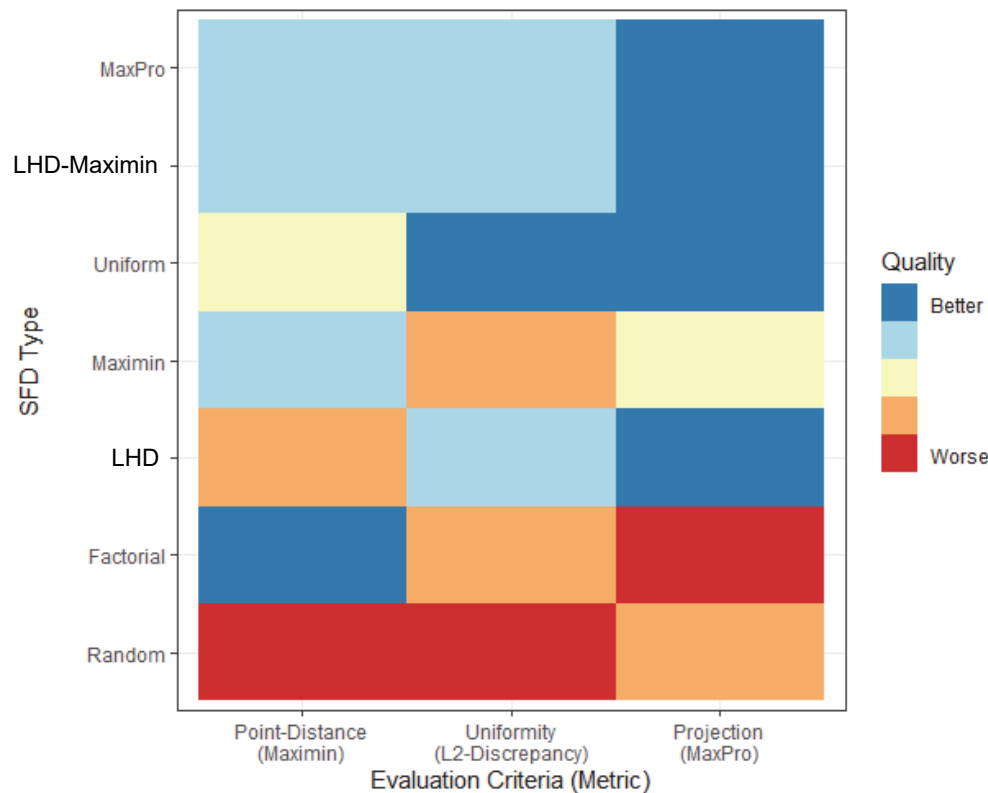
- Point-distance: Samples are placed as far apart from each other as possible. [**Maximin**]
- Uniformity: All regions of the design space are equally well-represented. [**Center  $L^2$  Discrepancy**]
- Projection: The design is robust to variables being collapsed. [**MaxPro**]



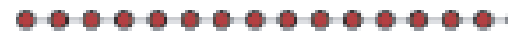
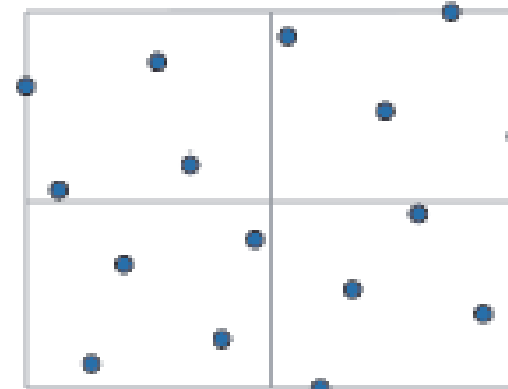


# I prefer maximin sliced Latin hypersquare designs (Maximin SLHD) and MaxPro SFDs

General recommendations:  
Maximin (Sliced) LHD or MaxPro



Latin Hypersquare



SFD – Space-Filling Design; SLHD – Sliced Latin Hypersquare Design

# I prefer maximin sliced Latin hypersquare designs (Maximin SLHD) and MaxPro SFDs

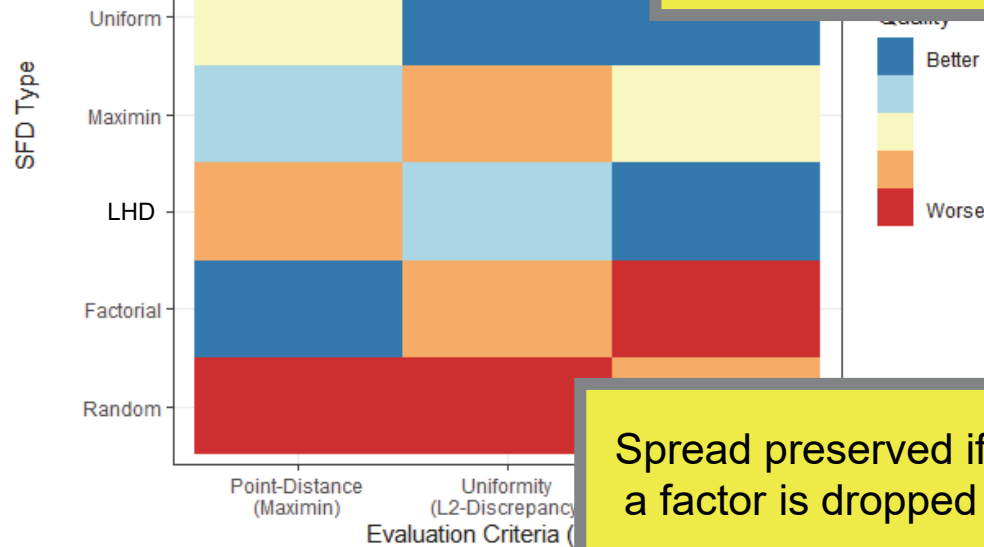
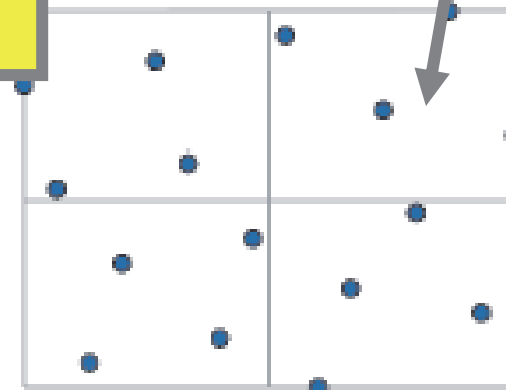
General recommendations:  
Maximin (Sliced) LHD or MaxPro

Good spread in the design space

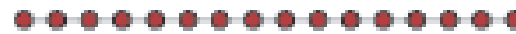
Handles a small (<5) number of categorical factors

Handles more categorical factors better, while also resembling maximin SLHD

Latin Hypersquare

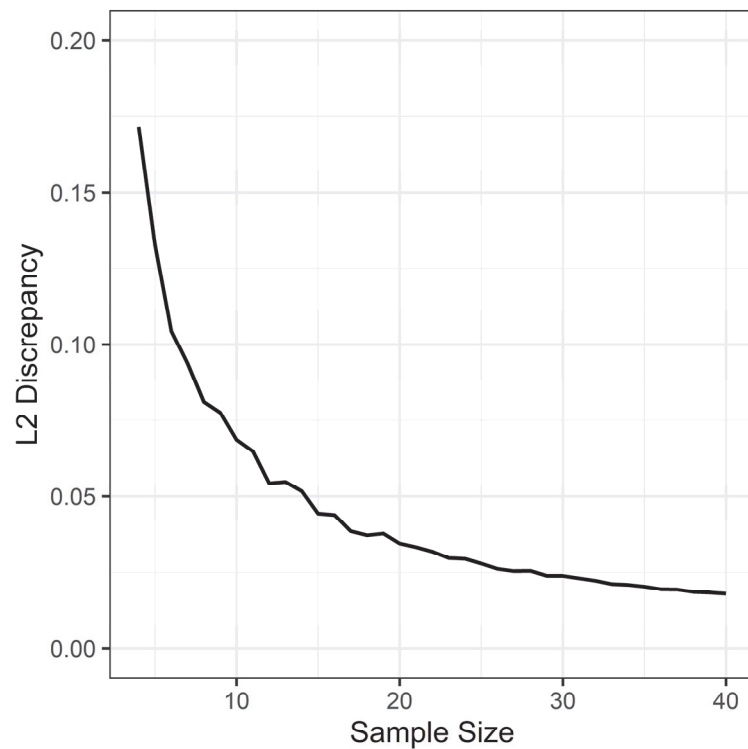


Spread preserved if a factor is dropped



# How should we choose the sample size when generating an SFD?

SCREE PLOT



$$n = 10d$$

## Choosing the Sample Size of a Computer Experiment: A Practical Guide

**Jason L. LOEPPKY**

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**Jerome SACKS**

National Institute of Statistical Sciences  
Research Triangle Park, NC 27709  
([sacks@niss.org](mailto:sacks@niss.org))

**William J. WELCH**

Department of Statistics  
University of British Columbia  
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([will@stat.ubc.ca](mailto:will@stat.ubc.ca))

We provide reasons and evidence supporting the informal rule that the number of runs for an effective initial computer experiment should be about 10 times the input dimension. Our arguments quantify two key characteristics of computer codes that affect the sample size required for a desired level of accuracy when approximating the code via a Gaussian process (GP). The first characteristic is the total sensitivity of a code output variable to all input variables; the second corresponds to the way this total sensitivity is distributed across the input variables, specifically the possible presence of a few prominent input factors and many impotent ones (i.e., effect sparsity). Both measures relate directly to the correlation structure in the GP approximation of the code. In this way, the article moves toward a more formal treatment of sample size for a computer experiment. The evidence supporting these arguments stems primarily from a simulation study and via specific codes modeling climate and ligand activation of G-protein.

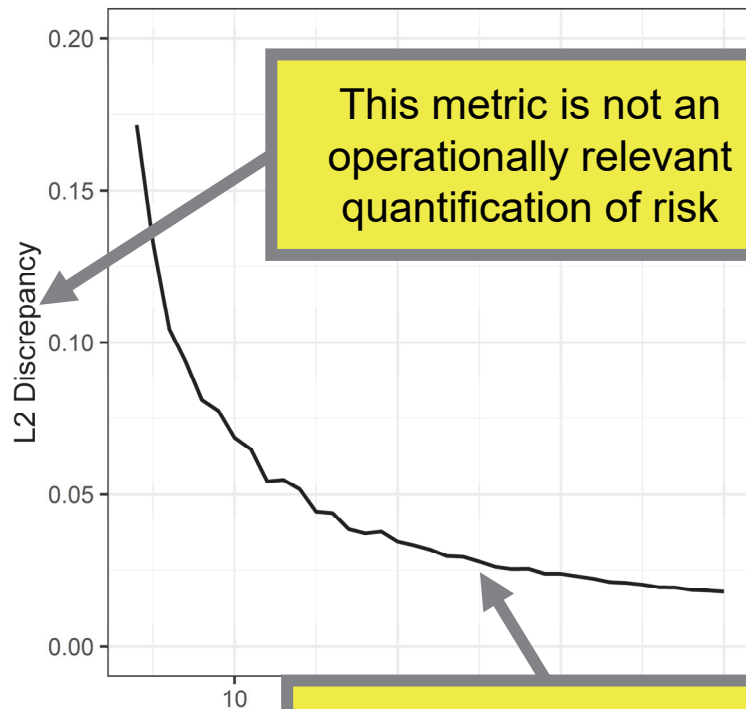
**KEY WORDS:** Curse of dimensionality; Effect sparsity; Gaussian process; Latin hypercube design; Prediction accuracy; Random function.

# How should we choose the sample size when generating an SFD?

Seems too small in stochastic cases

$$n = 10d$$

## SCREE PLOT



This metric is not an operationally relevant quantification of risk

Where is an objective cut-off?

Choosing the Sample Size of a Computer Experiment: A Practical Guide

Jason L. LOEPPKY  
Mathematics, Statistics, and Physics  
University of British Columbia, Okanagan  
Kelowna, BC V1V 1V7  
Canada  
(jason@stat.ubc.ca)

Jerome SACKS  
National Institute of Statistical Sciences  
Research Triangle Park, NC 27709  
(sacks@niss.org)

William J. WELCH  
Department of Statistics  
University of British Columbia  
Vancouver, BC V6T 1Z2  
Canada  
(will@stat.ubc.ca)

We provide reasons and evidence supporting why an initial computer experiment should be able to identify key characteristics of computer codes that are important when approximating the code via a Gaussian process. We discuss the effect of the number of input variables, the distribution of a code output variable to all input variables, and the effect of many input variables (i.e., effect sparsity) on the GP approximation of the code. In addition, we provide a sample size for a computer experiment. This is done via a simulation study and via specific codes.

KEY WORDS: Curse of dimensionality, Prediction accuracy, Ra

ADJACENT QUESTION:  
WHEN SHOULD  
REPLICATES BE USED  
(AND HOW MANY?)

The minimum may be all you get

What are the challenges in statistically validating a computer model?



# IDA publications and presentations introduce and recommend surrogate modeling best practices



INSTITUTE FOR DEFENSE ANALYSES

## Metamodeling Techniques for Verification and Validation of Modeling and Simulation Data

John T. Haman, Project Leader

Curtis G. Miller

September 2022

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IDA Paper P-33230

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<https://testscience.org>



## Space Filling Designs and Metamodeling for Understanding Modeling & Simulation Behavior

Curtis Miller

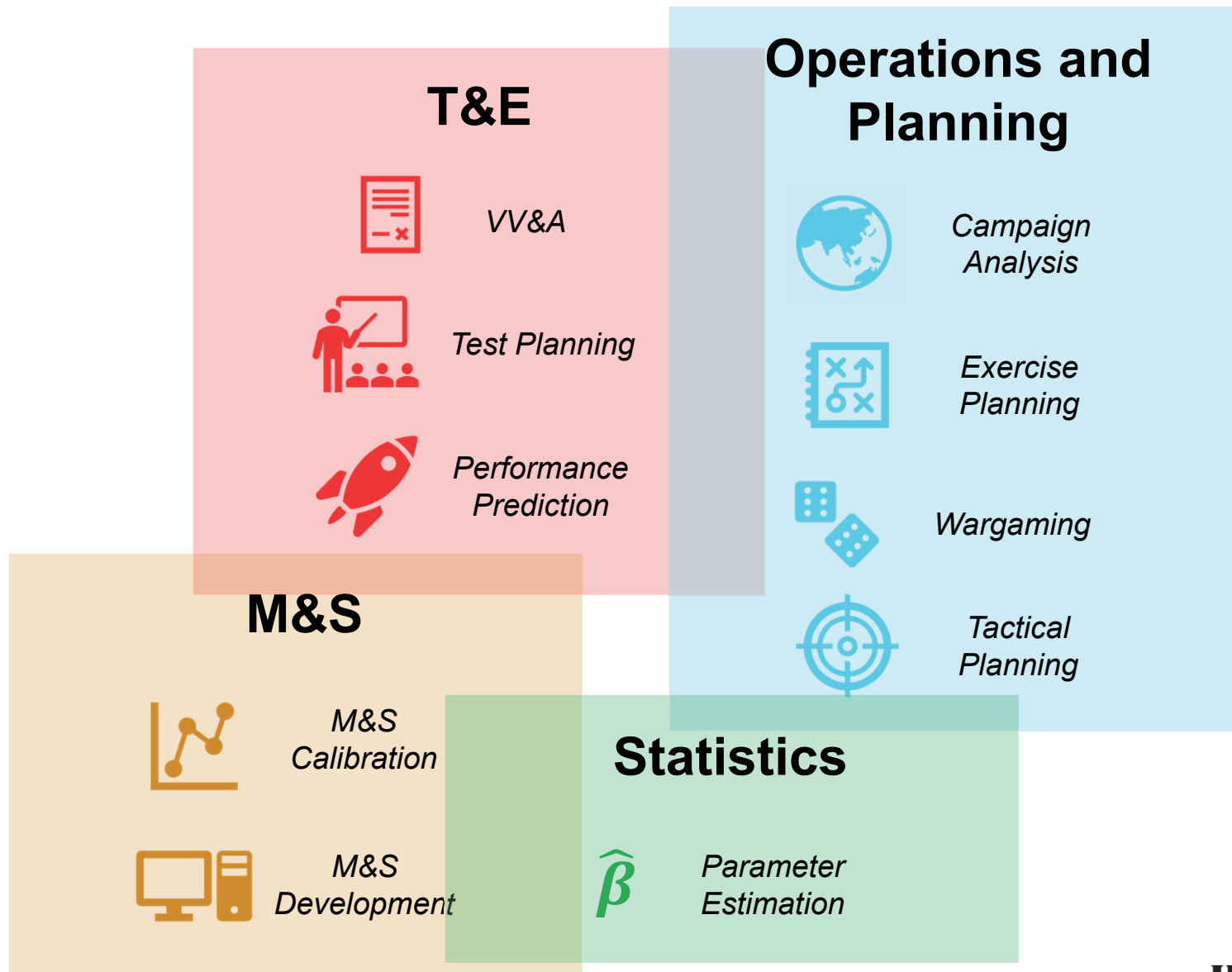
April 26, 2022

**Institute for Defense Analyses**  
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<https://dataworks.testscience.org/>



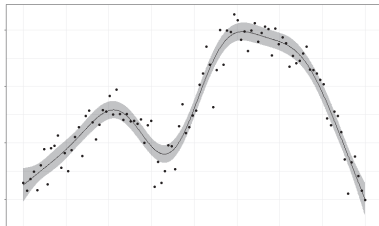
# A statistical surrogate is a useful product in and of itself



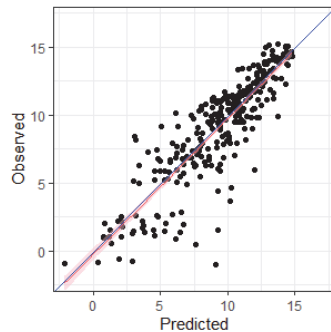
M&S – Modeling and Simulation; T&E – Test and Evaluation; VV&A – Verification, Validation, and Accreditation

# Statistical surrogates must allow for relevant assessments of M&S prediction quality

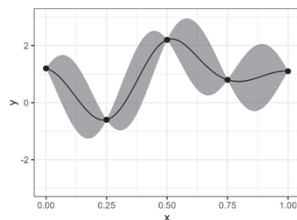
*Statistical surrogates should...*



*... discover unanticipated trends between factors and response variables*



*... predict observed responses well*

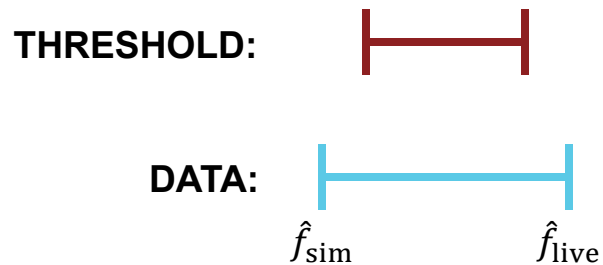


*... quantify uncertainty in M&S outputs*



# Statistical surrogates must allow for relevant assessments of M&S prediction quality

*Statistical surrogates should...*



*... allow comparing real-world outcomes to M&S predictions*



*... allow expert judgement on prediction quality*

# We recommend different statistical surrogate procedures based on the M&S output

	CONTINUOUS OUTPUT	DISCRETE OUTPUT
DETERMINISTIC	GAUSSIAN PROCESS (GP)	NEAREST NEIGHBOR (NN) DECISION TREE
STOCHASTIC	GENERALIZED ADDITIVE MODEL (GAM)	

# We demonstrate methods using a simple numerical ODE solver

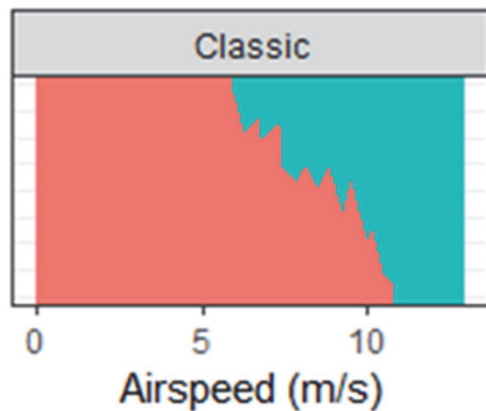
$$\dot{V} = -C_D (\rho V^2 / 2) S / m - g \sin(\gamma)$$

$$\dot{\gamma} = (C_L (\rho V^2 / 2) S / m - g \cos(\gamma)) / V$$

$$\dot{h} = V \sin(\gamma)$$

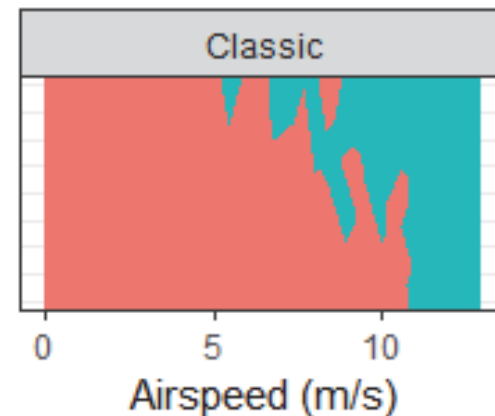
$$\dot{r} = V \cos(\gamma)$$

Randomize initial conditions for random output (i.e., variance in toss)



Loop ■ FALSE ■ TRUE

**DETERMINISTIC**

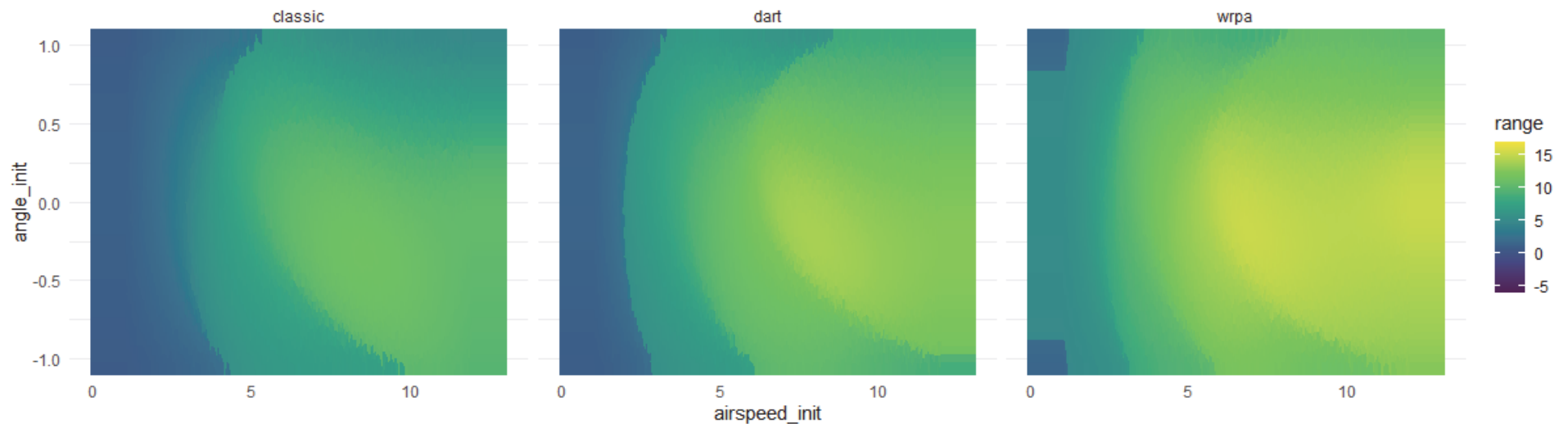


Loop ■ FALSE ■ TRUE

**STOCHASTIC**

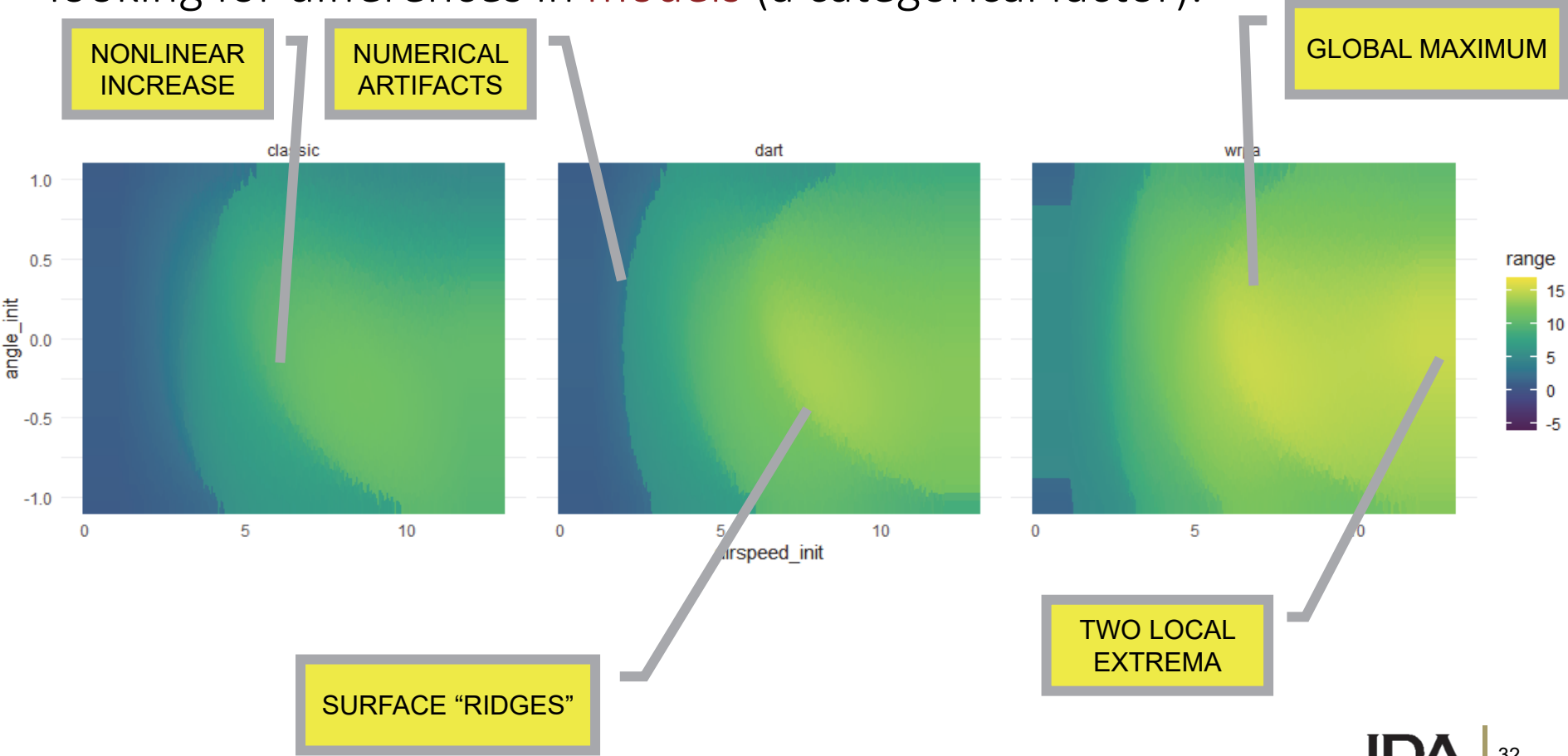
# Model, Airspeed, and Angle Study

Consider a study of the effects of initial **airspeed** and **angle** of flight (continuous factors) on flight terminal **range** (continuous), while also looking for differences in **models** (a categorical factor).



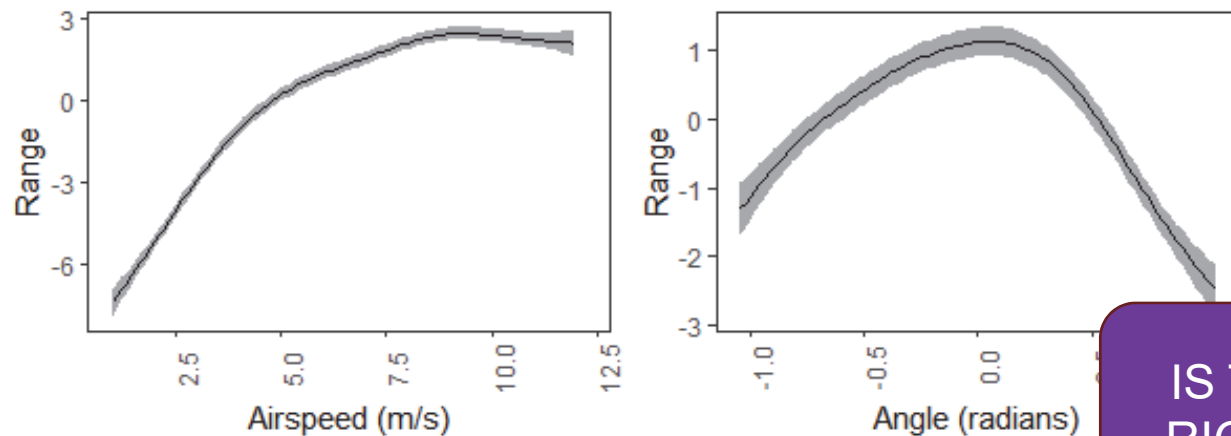
# Model, Airspeed, and Angle Study

Consider a study of the effects of initial **airspeed** and **angle** of flight (continuous factors) on flight terminal **range** (continuous), while also looking for differences in **models** (a categorical factor).



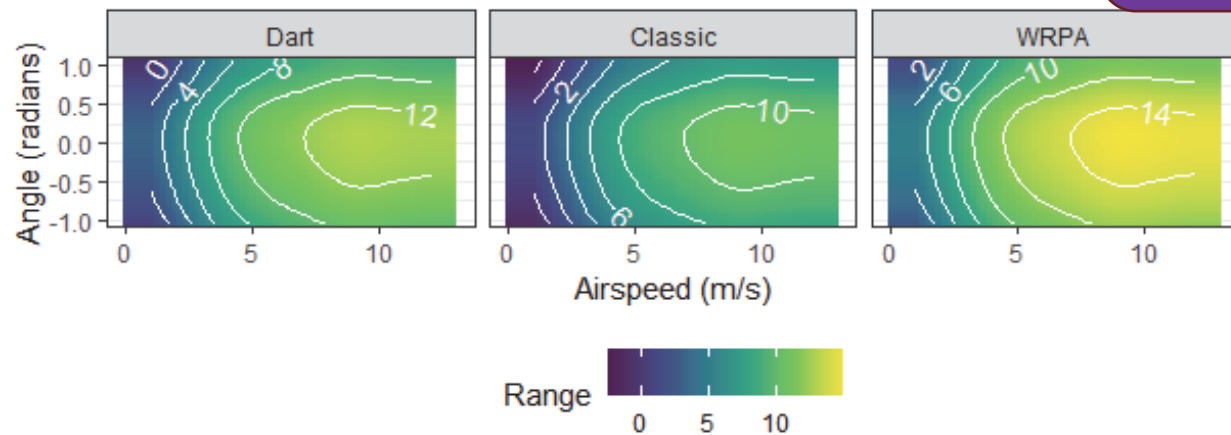
# GAMs allow for qualitative assessment of simulation system performance

$$\text{range}_i = \beta_0 + \beta_{cl}\text{classic}_i + \beta_{wr}\text{wrpa}_i + f_{as}(\text{airspeed}_i) + f_{an}(\text{angle}_i) + \epsilon_i$$

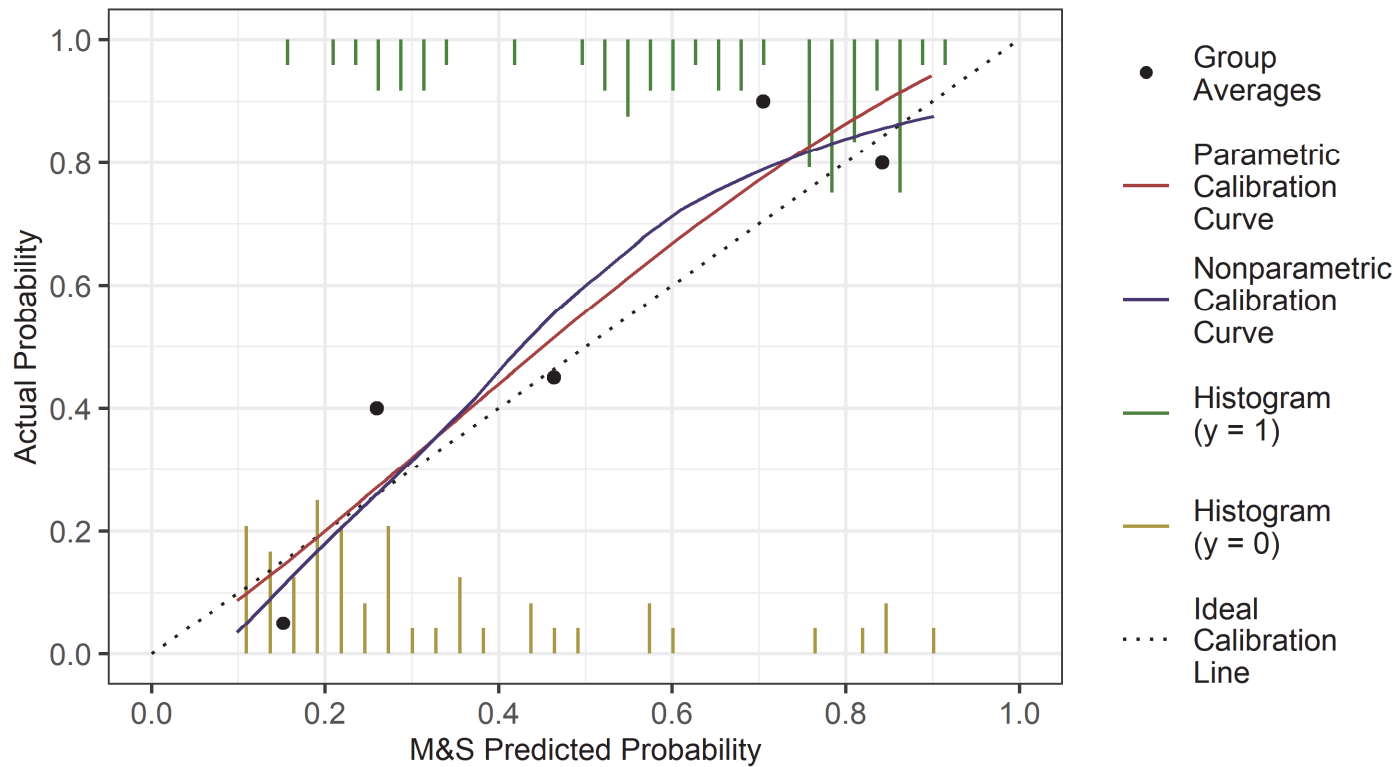


Average Range:  
 Dart: 9.3 m  
 Classic: 7.4 m  
 WRPA: 11.3 m

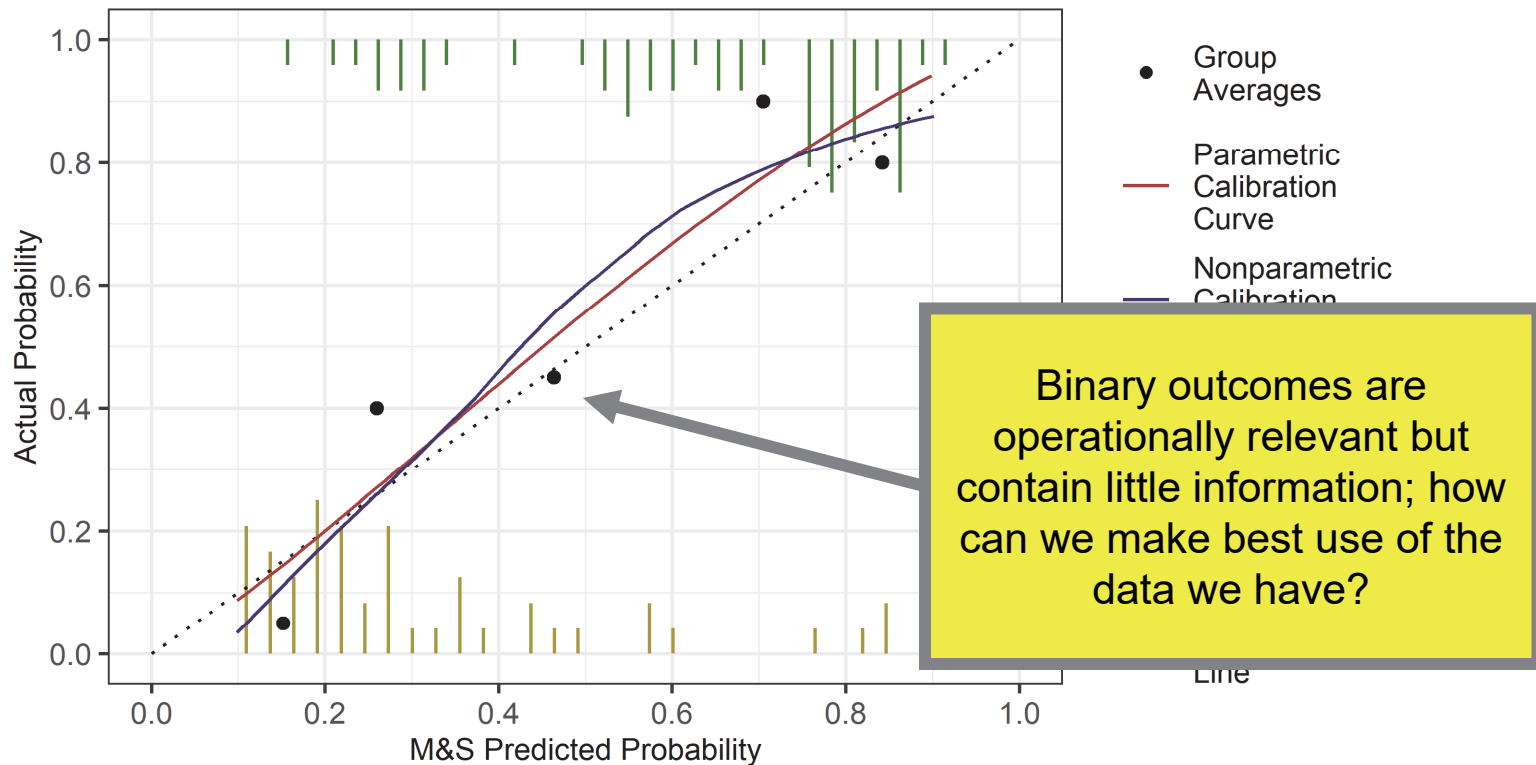
IS THIS RIGHT?



# How can statistical surrogates allow for M&S validation with small sample sizes?



# How can statistical surrogates allow for M&S validation with small sample sizes?





# How should statistical testing incorporate statistical surrogates?

$H_0$ : Sim=Live

$H_A$ :  $H_0$  is false

$$W = \frac{(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})^2}{\text{Var}(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})}$$

**THRESHOLD:**



**DATA:**



Reject if  $W > C_\alpha$ .

# How should statistical testing incorporate statistical surrogates?

$H_0$ : Sim=Live

$H_A$ :  $H_0$  is false

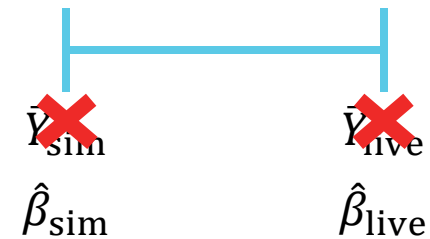
$$W = \frac{(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})^2}{\text{Var}(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})}$$

$$W = (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})^T V^{-1} (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})$$

THRESHOLD:



DATA:



Reject if  $W > C_\alpha$ .

# How should statistical testing incorporate statistical surrogates?

$H_0$ : Sim=Live

$H_A$ :  $H_0$  is false

~~$$W = \frac{(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})^2}{\text{Var}(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})}$$~~

~~$$W = (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})^T V^{-1} (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})$$~~

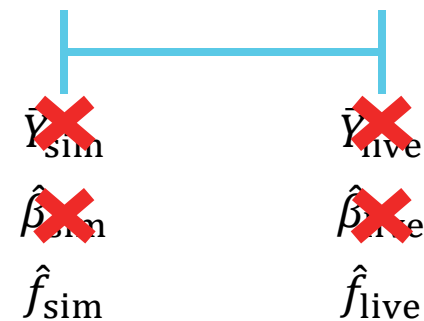
$$W = \|\hat{f}_{\text{sim}} - \hat{f}_{\text{live}}\|_{V^{-1}}^2$$

Reject if  $W > C_\alpha$ .

THRESHOLD:



DATA:



# How should statistical testing incorporate statistical surrogates?

$H_0$ : Sim=Live

$H_A$ :  $H_0$  is false

~~$$W = \frac{(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})^2}{\text{Var}(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})}$$~~

~~$$W = (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})^T V^{-1} (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})$$~~

$$W = \|\hat{f}_{\text{sim}} - \hat{f}_{\text{live}}\|_{V^{-1}}^2$$

Reject if  $W > C_\alpha$ .

THRESHOLD:



DATA:



Should these be

- Regression model coefficients?
- Response functions (e.g., GAM smooth)?
- Average response at select factor combinations?

How will we overcome the challenges of modeling and simulation validation?

# M&S validation faces many challenges that must be overcome

*T&E workforce  
statistical literacy*

*Turning policy into  
statistical practice*

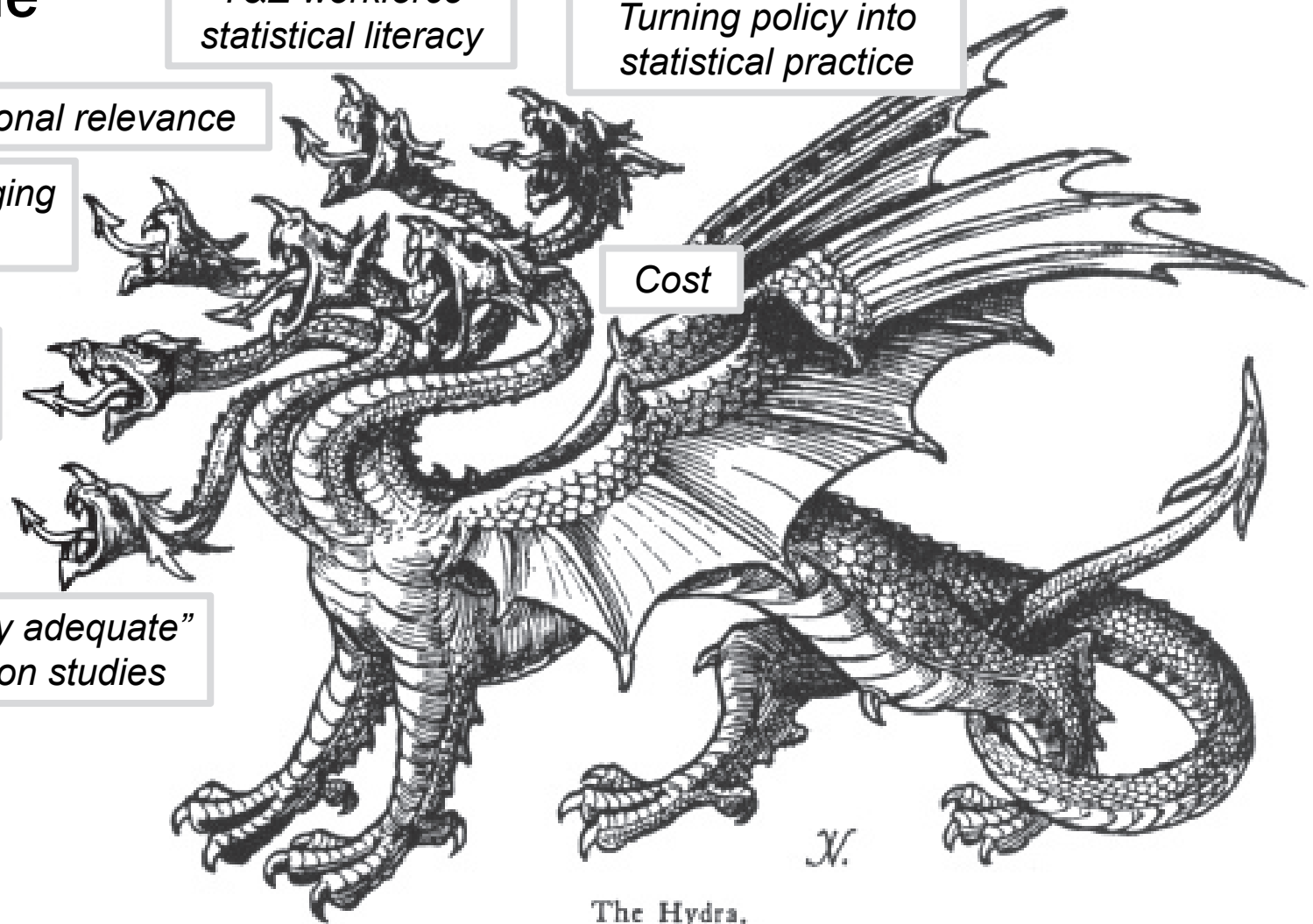
*Operational relevance*

*Continually changing  
M&S systems*

*Small real world  
samples*

*Cost*

*“Minimally adequate”  
simulation studies*



The Hydra.

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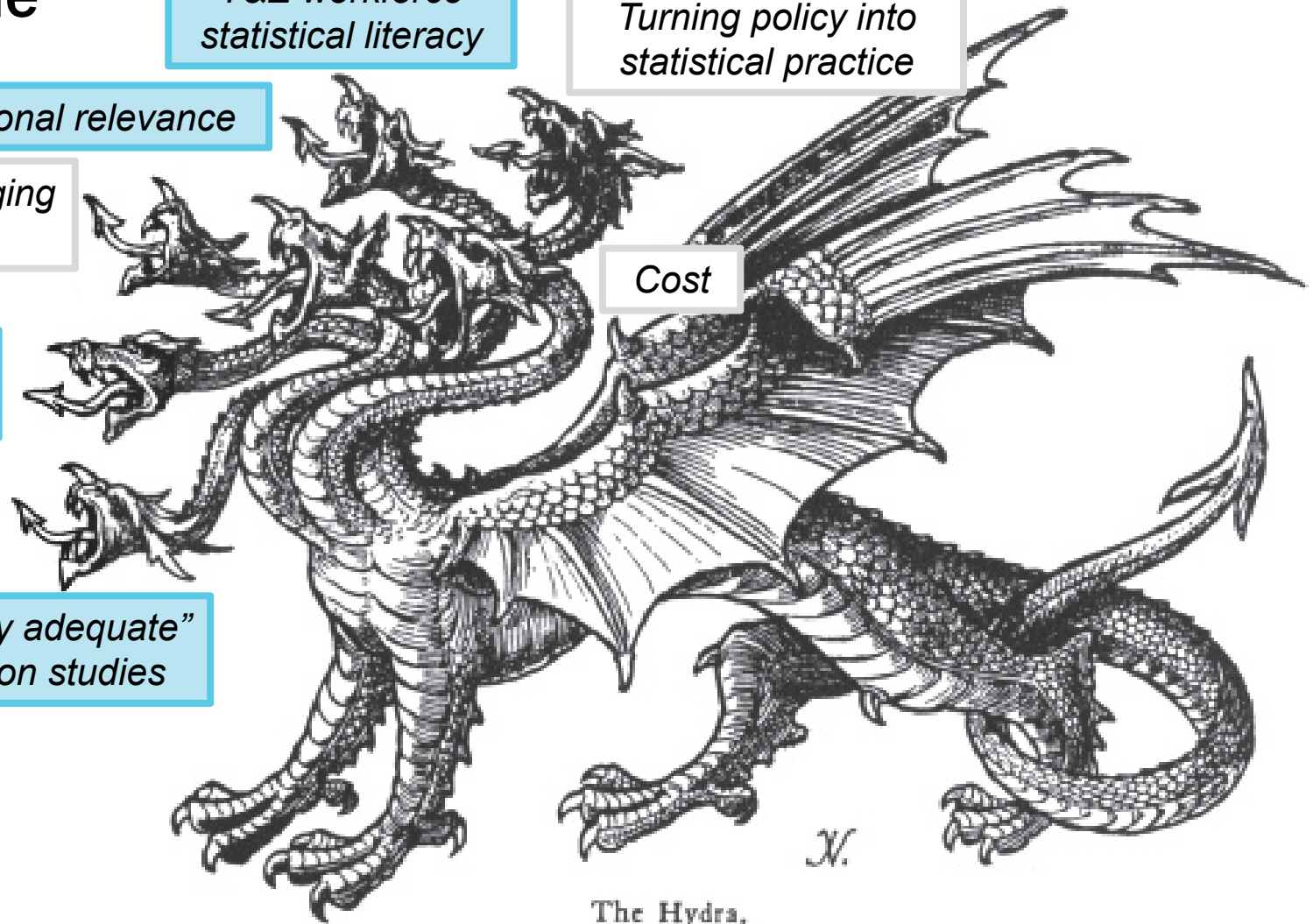
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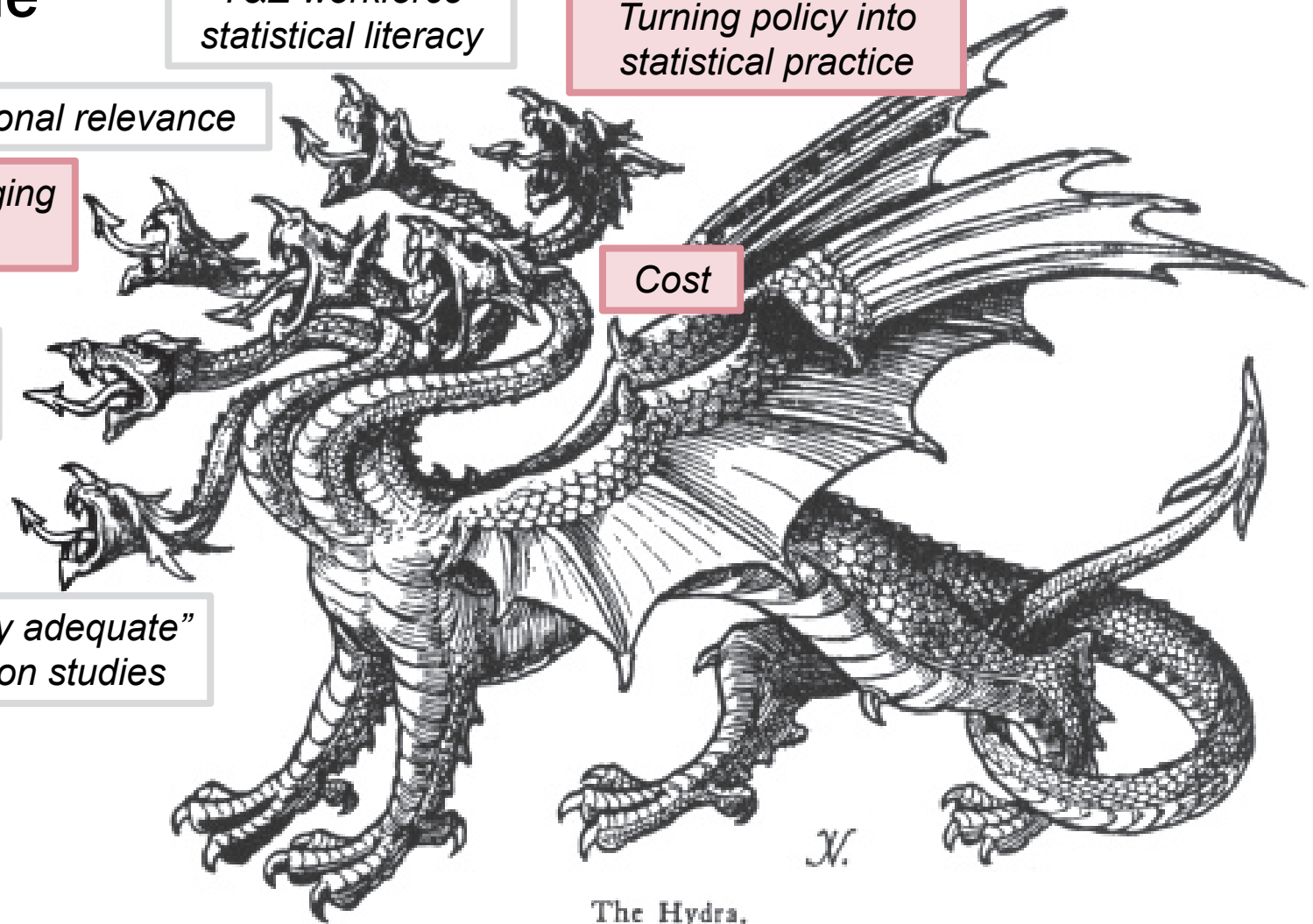
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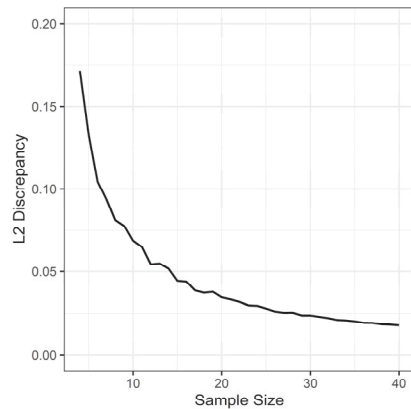
*“Minimally adequate”  
simulation studies*



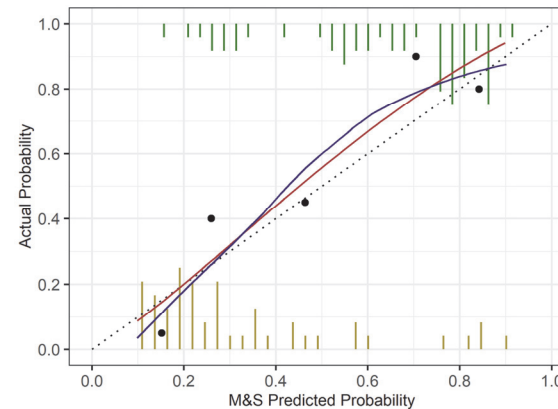
The Hydra.



# We want statisticians to address...



*... how many observations to collect from M&S*



*... how to best handle binary responses*

$$W = \|\hat{f}_{\text{sim}} - \hat{f}_{\text{live}}\|_{V^{-1}}^2$$

*... how to make statistical decisions with surrogate models*

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