SCIENTIFIC TEST & ANALYSIS TECHNIQUES
CENTER OF EXCELLENCE

Reducing Test to Purpose

Anthony Sgambellone
STAT COE
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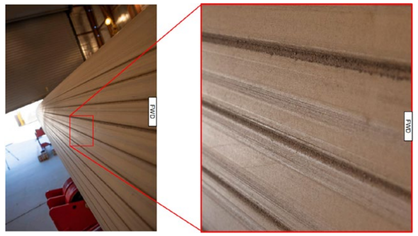
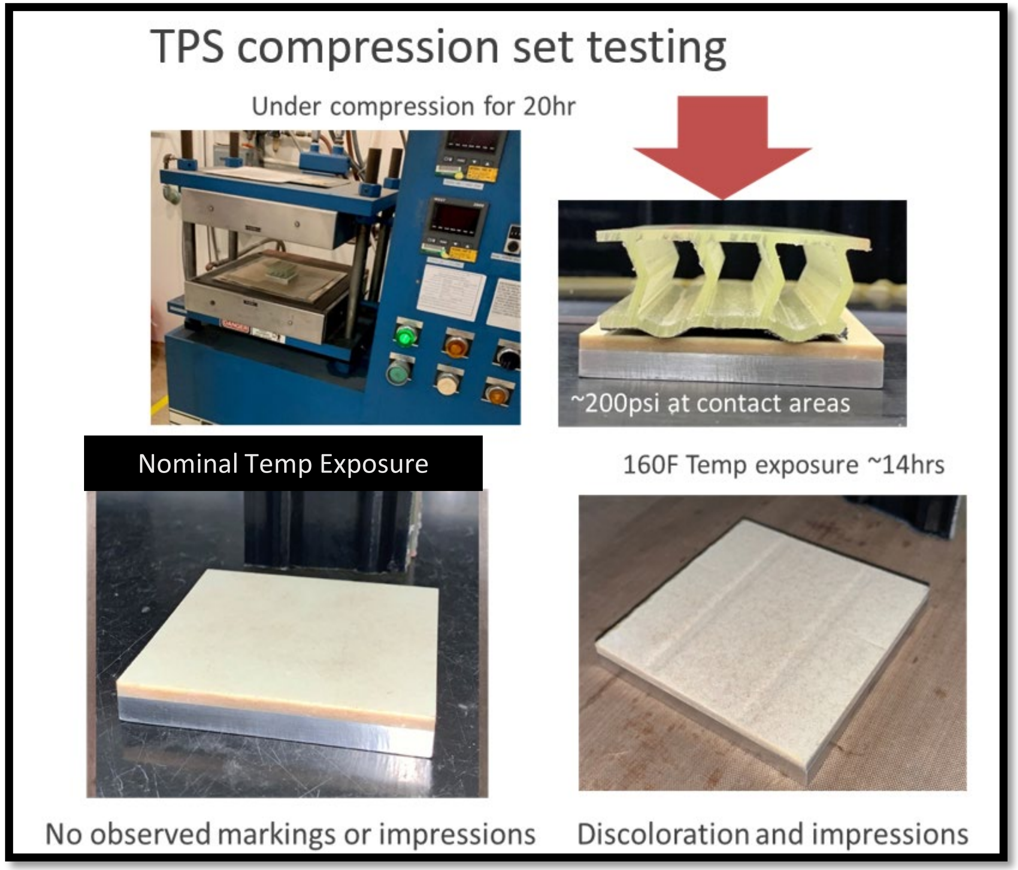
Problem – Striations in Missile Body Coating



- Protective coating striations following a launch test
- Striations aligned with launch pads

Root Cause Analysis

- Environmental Exposure
  - Test shot was exposed to >150°F before execution
  - This is greater than the glass transition temperature
- Friction during eject event
- Interface during storage
  - The material creeps slowly over time with applied pressure



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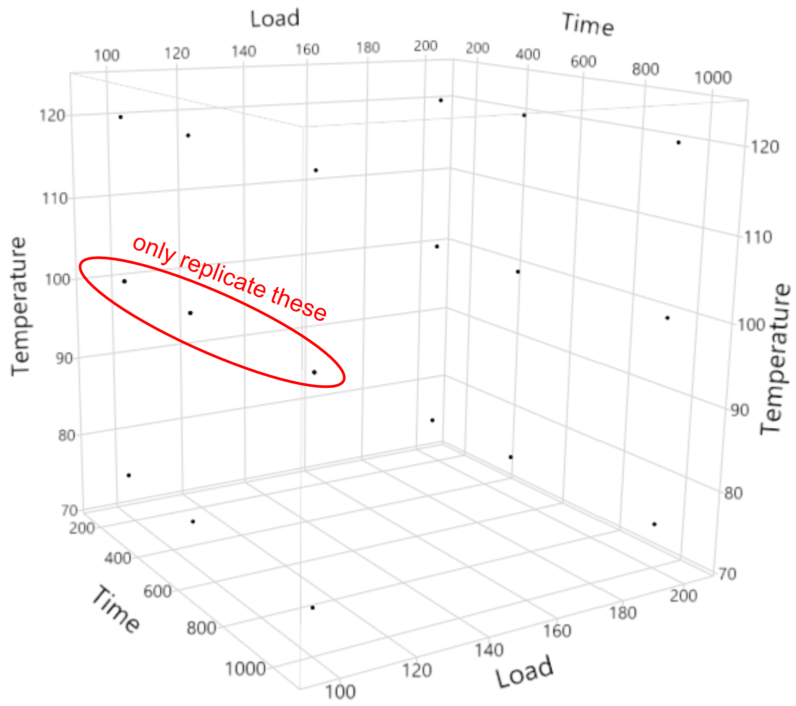
Initial Test Plan

- Test team proposed a 3x replicated full-factorial, but were concerned about required resources
- Factors:
  - Temperature: 75°, 100°,120°
  - Load: 100 psi (nominal), 200 psi (3σ)
  - Time: 170 hrs, 500 hrs, 1000 hrs



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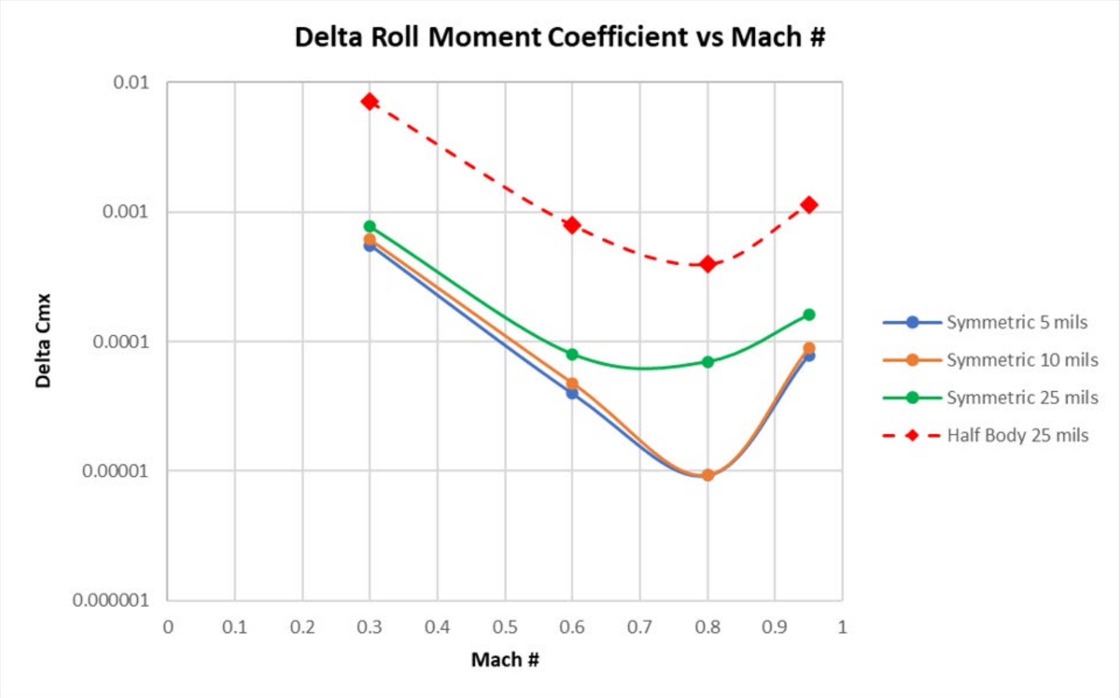
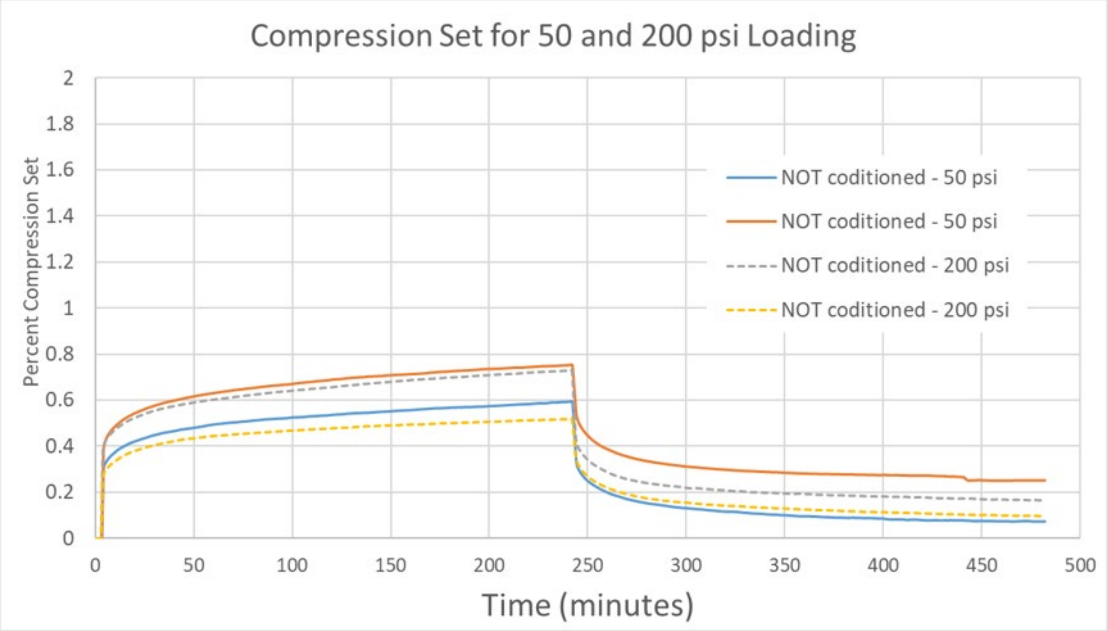
Prior Data Analysis: Estimating SNR

Aerodynamic simulations indicated minimal effect on performance for up to 10 mil striation depths



Preliminary tests were used to estimate standard deviation (0.5 mils)



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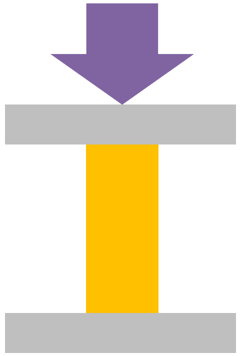
$$\mathbf{SNR \approx 20}$$

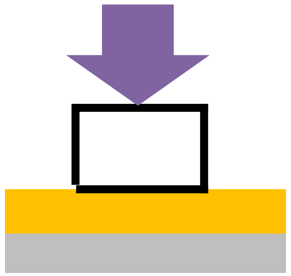
Modified Creep Testing Set Up

The test team conceived three set ups:

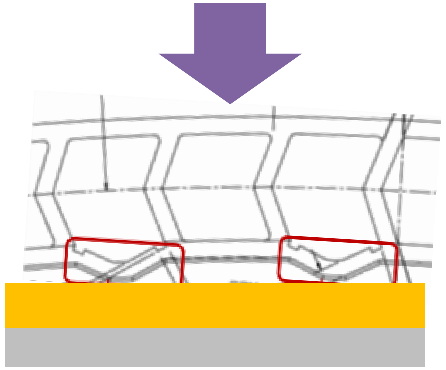
Test Case 1:
- Compression of coating only
- Standard compression test used to build material models



Test Case 2:
- Load applied directly to coating
- Used to validate TC1 and identify any differences due to geometry



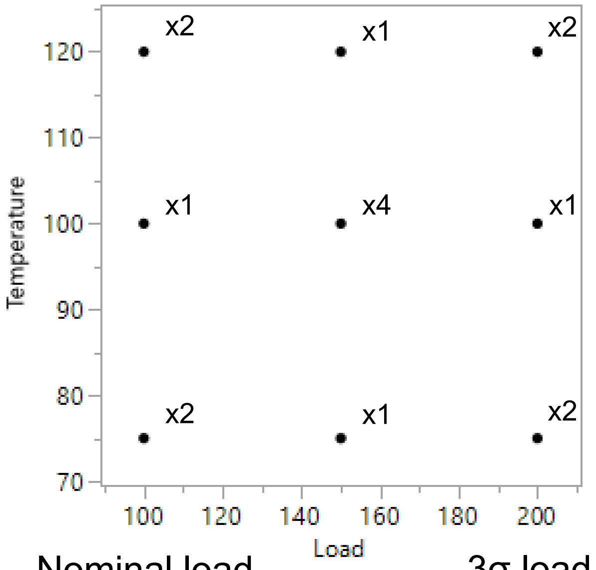
Test Case 3:
- Load applied to pad coupon
- Closest to real-life



Design: Test Case 1

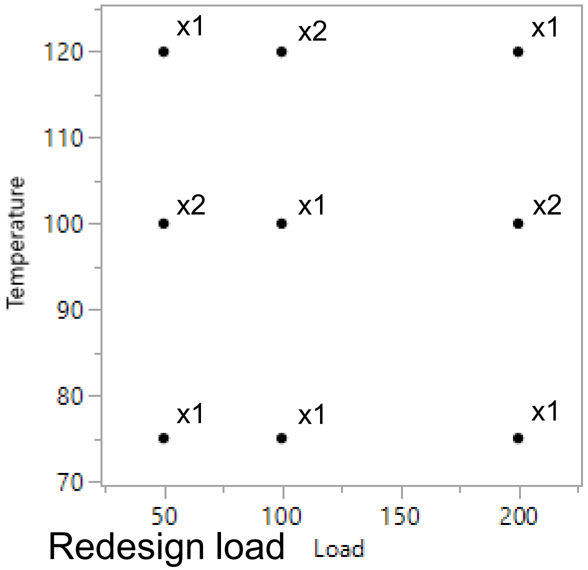
- Test set up allowed compression to be continuously measured over time (Only two factors: temperature and pressure)

Original Design:
16 runs
Load: 100, 150, 200 psi



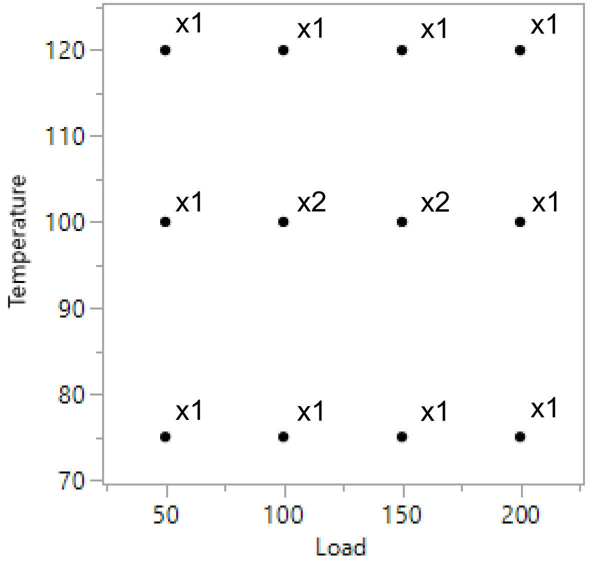
Nominal load 3σ load

Option 2:
13 runs
Load: 50, 100, 200 psi



Redesign load

Option 3:
14 runs
Load: 50, 100, 150, 200 psi



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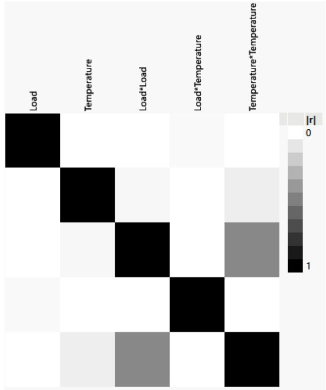
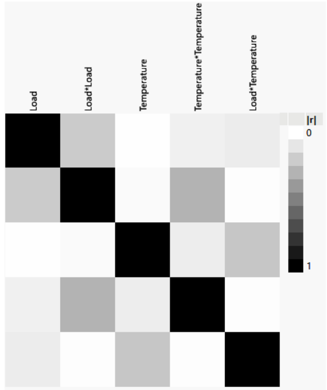
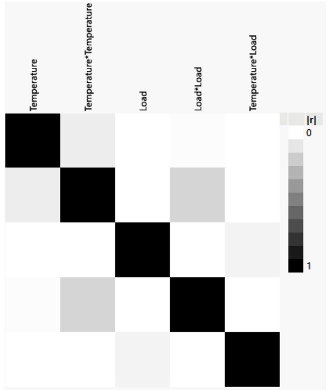
Design: Test Case 1

Design #	1	2	3
Software Package	JMP	JMP	JMP
Name/Design Type	Original	D-Optimal	D-Optimal
Factor	Load	Load	Load
Levels	3 (100,150,200)	3 (50,100,200)	4 (50,100,150,200)
Factor	Temp	Temp	Temp
Levels	3	3	3
Model Supported	ME, 2FI, Q	ME, 2FI, Q	ME, 2FI, Q
Signal to Noise Ratio	20.0	20.0	20
Confidence	0.95	0.95	0.95
# Center Points	4	1	4*
Total Runs	16	13	14
Power for ME @ S/N (>0.80)	1	1	1
Power for 2FI @ S/N (>0.80)	1	1	1
FDS Pred Err @50% (<1.0)	0.21	0.33	0.23
FDS Pred Err @95% (<1.0)	0.36	0.52	0.44
Aliasing	none	none	load is aliased with cubic load term
Color Correlation			

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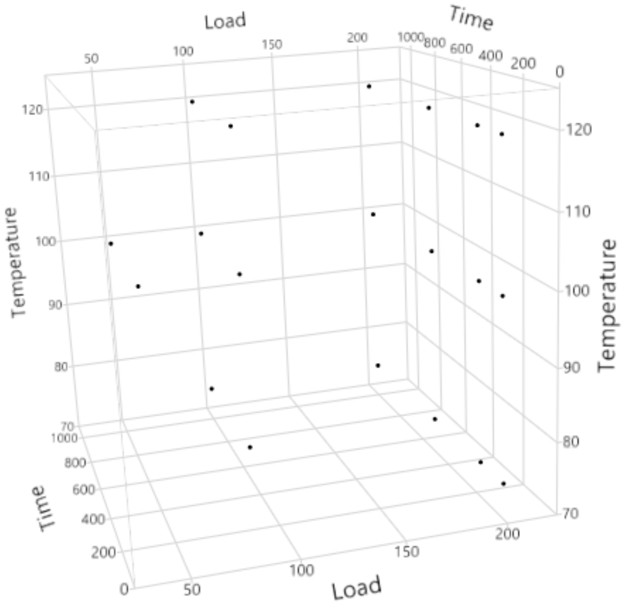
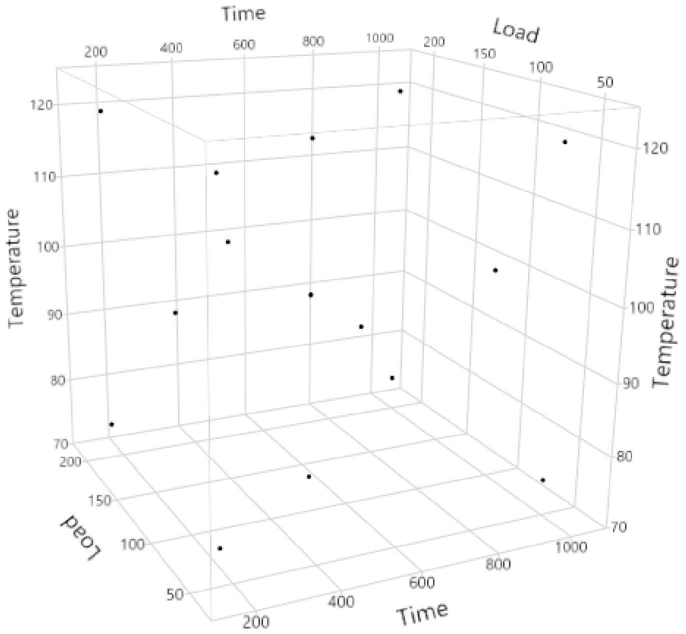
Design: Test Case 3

- Compression could not be continuously measured over time

Original Design:
20 runs



Option 2:
16 runs



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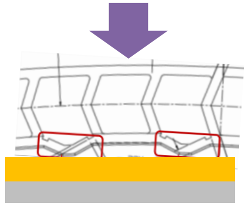
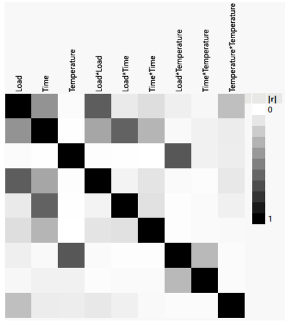
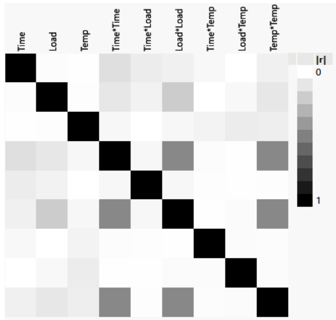
Design: Test Case 3



Design #	1	2
Software Package	JMP	JMP
Name/Design Type	Original	D-Optimal
Factor	Load	Load
Levels	3 (50,100,200)	3 (50,100,200)
Factor	Temp	Temp
Levels	3	3
	Time	Time
	4	3
Model Supported	ME, 2FI, Q	ME, 2FI, Q
Signal to Noise Ratio	20.0	20.0
Confidence	0.95	0.95
# Center Points	5*	2*
Total Runs	20	16
Power for ME @ S/N (>0.80)	1	1
Power for 2FI @ S/N (>0.80)	1	1
FDS Pred Err @50% (<1.0)	0.40	0.35
FDS Pred Err @95% (<1.0)	2.00	0.58
Aliasing	none	none
Color Correlation		

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Analysis

- Each test unit was measured for change in thickness over a two-dimensional space, for a three-dimensional response surface.
- Due to the shape of the pad there were two linear regions with striations of interest.
- To simplify the analysis, we used the maximum depth of each striation and modeled the left and right striations separately.

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Response

- Percent of Creep in the protective material was expected to be a more consistent and generalizable response than raw depth of the striation, so that is the metric that was used for modeling
  - The results of the model could then be transformed back to standard units given thickness for comparison to meaningful thresholds

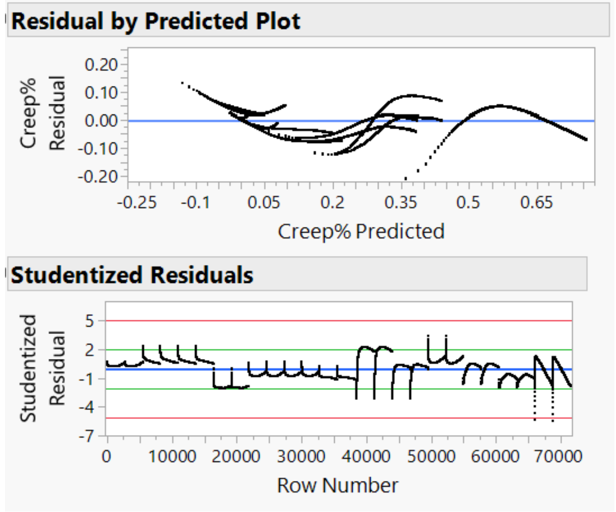
TC1 Modeling

- Correlated measurements
  - Accounting for the correlation between measurements of the same coupon over time would be ideal, but an effective prediction may still be achieved with a lesser investment of time
- Regression
  - Independence assumption is violated, and probably others
    - Most assumptions for regression apply only to the calculation of statistical significance: it may still provide the Best Linear Unbiased Estimator for prediction to be evaluated post-hoc as many other machine learning algorithms
- Neural Networks
  - Can model any shape of relationship
  - More difficult to optimize

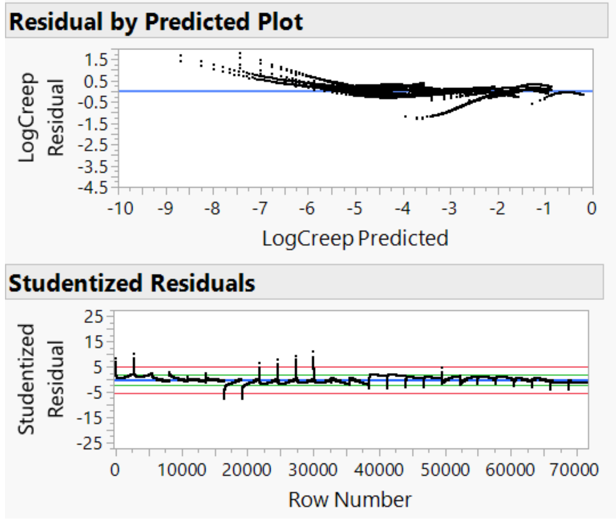
Transformation for Linear Fit

- Test case 1 data was available first, and provided valuable insight into later analyses
- From the shape of the relationship between Time and Creep in the raw units, a log Time relationship seemed reasonable
- Log Time was an improvement, but there was still a departure from linearity that was improved by a log transformation of Creep in context of the rest of the model

Ln Time

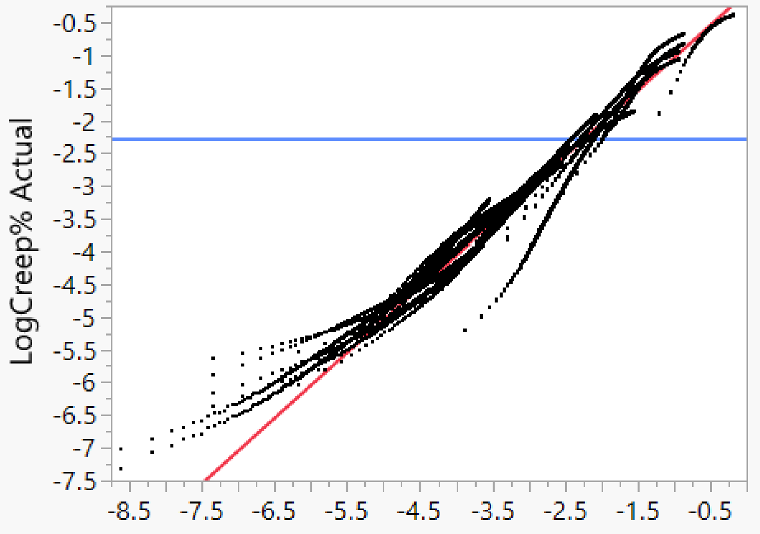


Ln Time and Ln Creep



Regression

- Meaningfully interpretable
- Restricted in form, unable to fully capture shape over time
- Still captures >98% of variation in TPS creep
- High creep scenarios appear to have a different shape.
  - A limitation of standard linear regression application is the inability to fit a different transformation to maintain relative linearity at different factor levels



LogCreep% Predicted RMSE=0.1662
RSq=0.98302 PValue=<.0001

Term	Estimate
Intercept	-10.00233
PSI	0.0137661
Temp	0.053794
LogTime	0.249108
(PSI-115.564)*(PSI-115.564)	-9.042e-5
(PSI-115.564)*(Temp-98.4966)	3.3953e-5
(Temp-98.4966)*(Temp-98.4966)	-0.000188
(PSI-115.564)*(LogTime-4.96091)	-0.000676
(Temp-98.4966)*(LogTime-4.96091)	-0.007119
(LogTime-4.96091)*(LogTime-4.96091)	-0.013964

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Artificial Neural Network

- Able to capture the shape over time much better than regression.
  - Only the most extreme condition seems to have additional curvature: 200 PSI, 120 degrees

Training

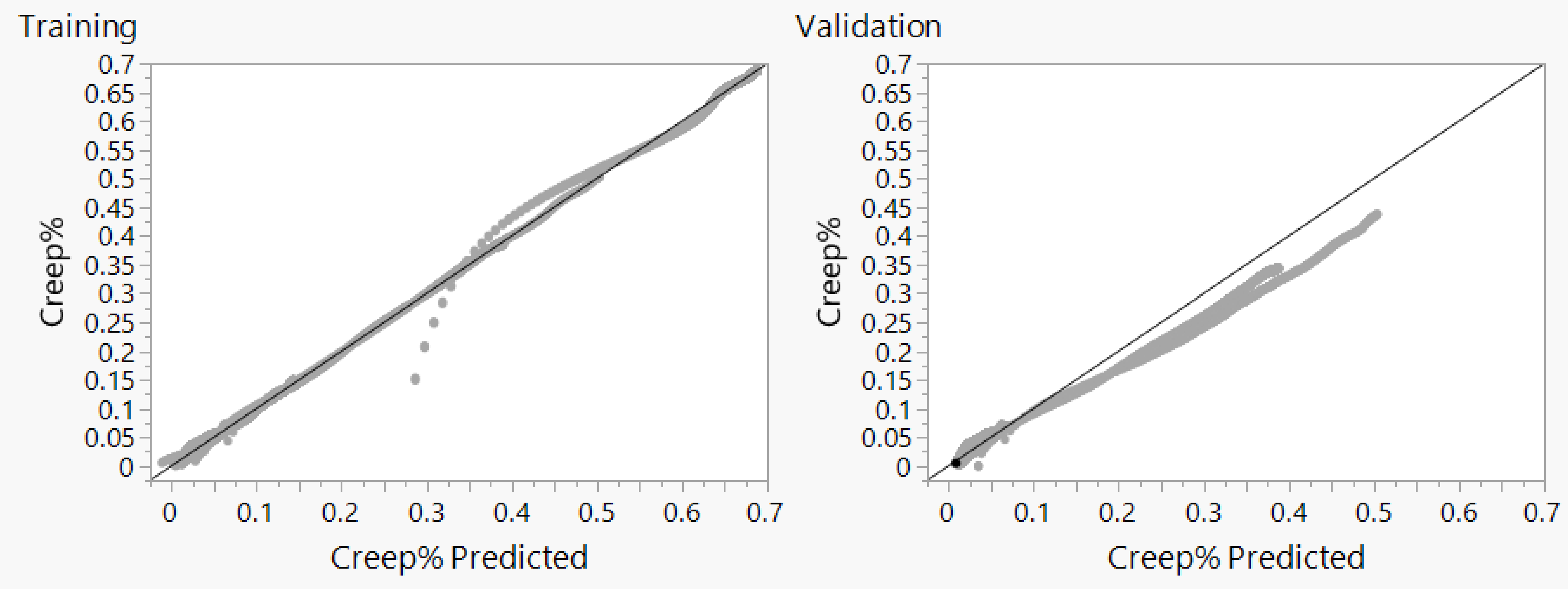
Creep%

Measures	Value
RSquare	0.9996925
RMSE	0.0038309
Mean Abs Dev	0.0026033
-LogLikelihood	-193339.1
SSE	0.6844367
Sum Freq	46636

Validation

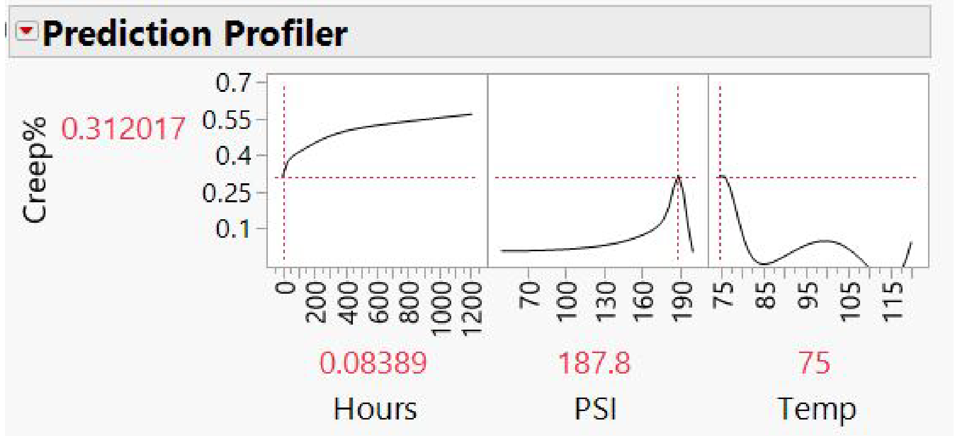
Creep%

Measures	Value
RSquare	0.9391306
RMSE	0.037518
Mean Abs Dev	0.0272683
-LogLikelihood	-46115.25
SSE	34.824057
Sum Freq	24740



ANN: Interpolation Concerns

- The artificial neural network model may not be appropriate for interpolation with non space-filling data. Unlike regression models, artificial neural networks do not necessarily fit according to a well-behaved relationship between observed points.
- Consider the profiler plots below. There was no data between 100 and 200 psi, so the model did not need to constrain predicted values in that region. At approximately 0 time and 75 degrees, the model would predict significant creep at 190 psi while appropriately predicting near 0 Creep at observed pressures.



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TC3 Analysis: Multivariate Regression

- The maximum depth of each striation was taken, and a multivariate analysis performed in JMP. The joint evaluation of effects is below

Source	LogWorth		PValue
Temp	7.303		0.00000
PSI	5.022		0.00001
PSI*PSI	4.919		0.00001
PSI*Temp	3.350		0.00045
LogTime	2.868		0.00136
Temp*Temp	2.205		0.00623
Temp*LogTime	1.944		0.01138

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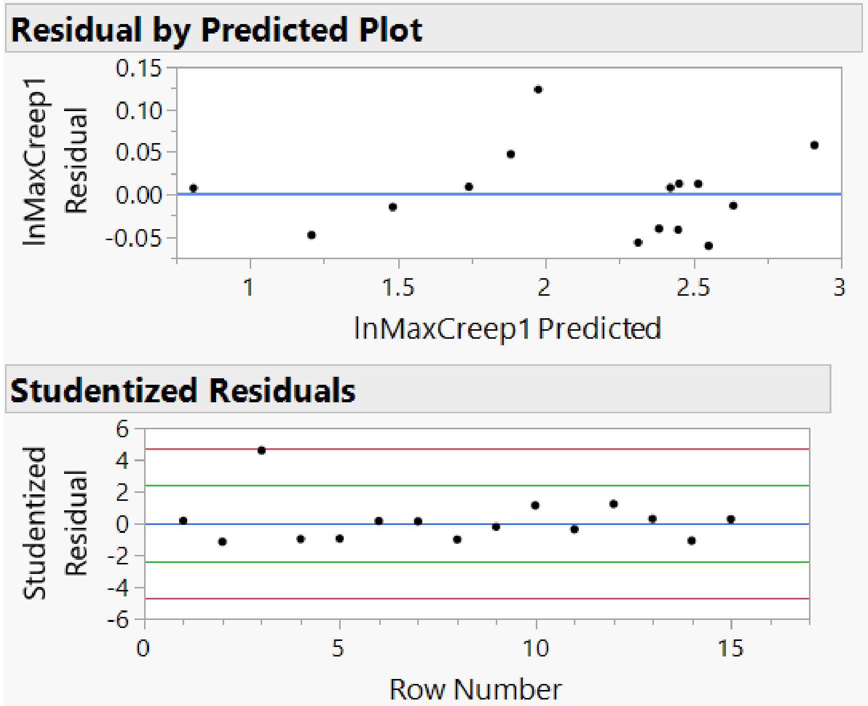
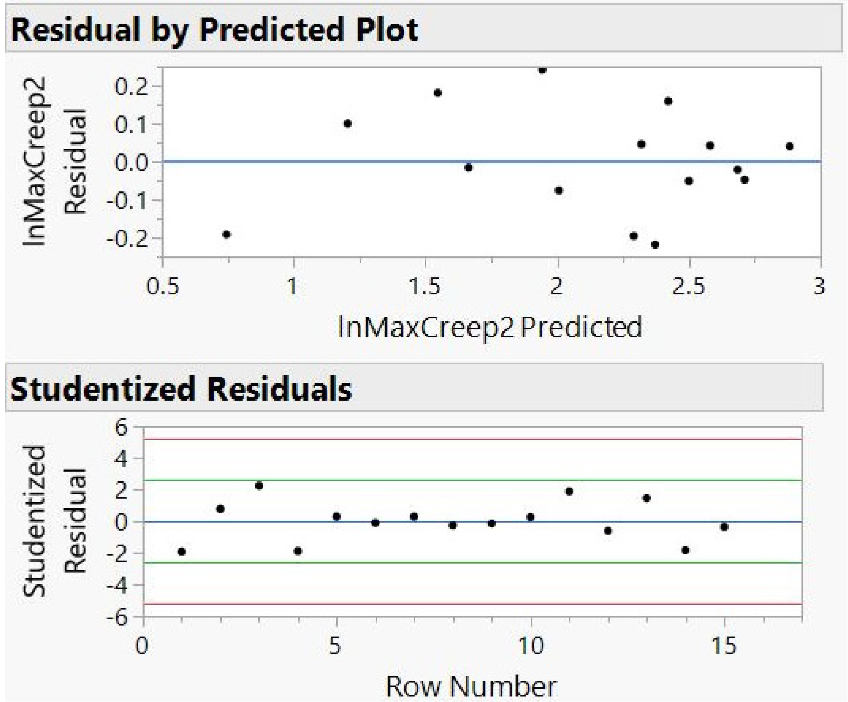
Multivariate

- The model fitting the second striation appeared to be a better fit for the distribution of errors

Striation 1 Model



Striation 2 Model



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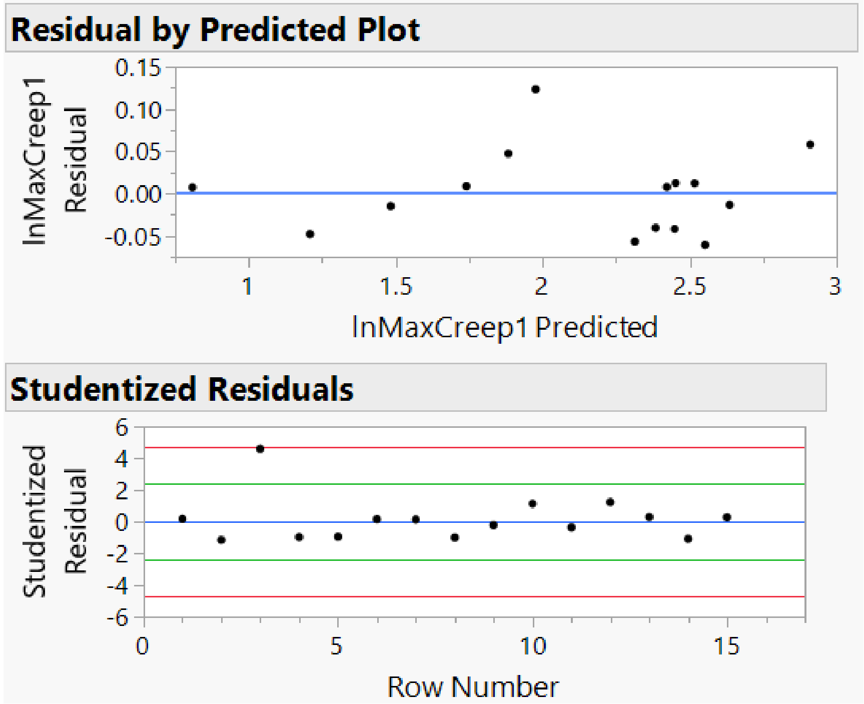
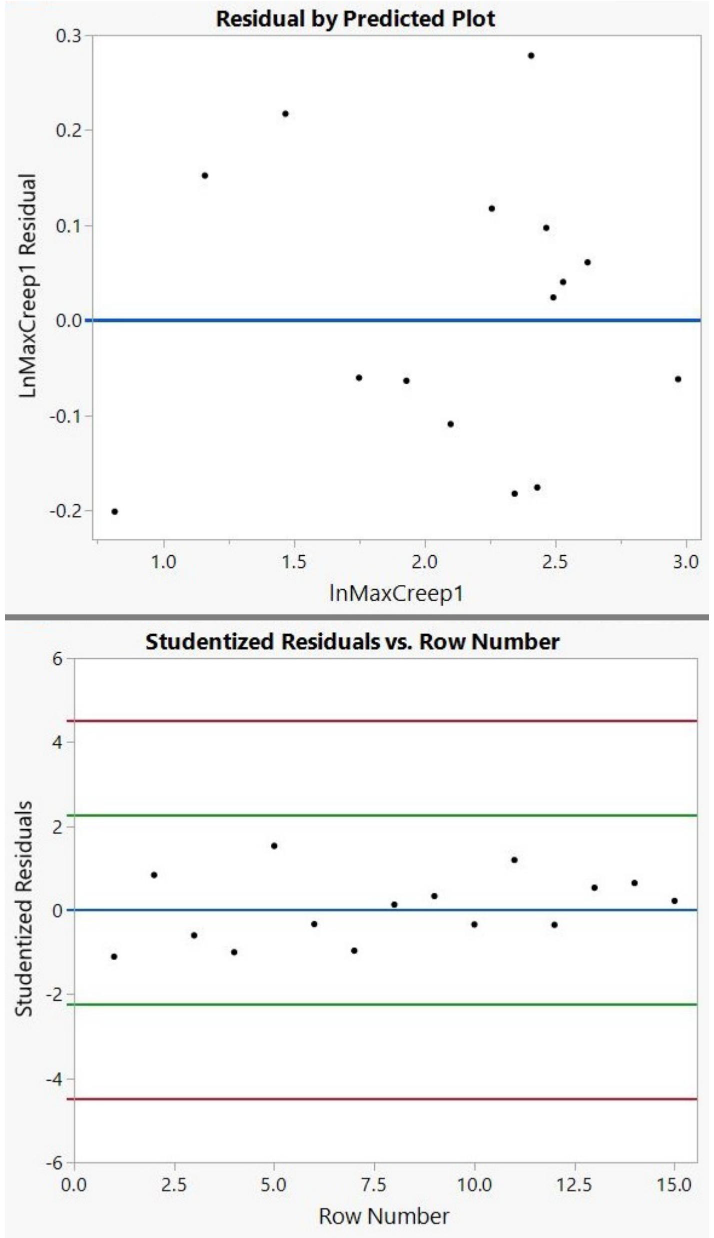
Multivariate

- The model from striation 2 seemed to provide a more robust fit even on the values of striation 1

Model 1 on Striation 1



Model 2 on Striation 1



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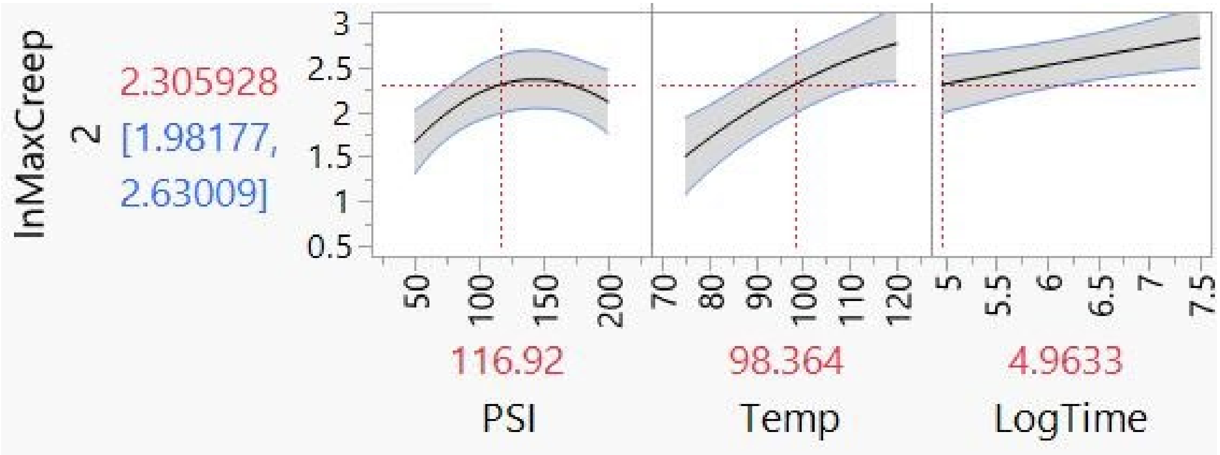
Final Model

- The quadratic effect of Temperature and the interaction between Temperature and Time remain in model 2 because they appeared significant in the fit of model 1, but they have little effect in comparison to the noise.

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-1.710921	0.533909	-3.20	0.0150*
PSI	0.0043639	0.00098	4.45	0.0030*
Temp	0.0251985	0.002819	8.94	<.0001*
LogTime	0.2071077	0.069762	2.97	0.0208*
(PSI-116.667)*(PSI-116.667)	-8.036e-5	2.388e-5	-3.37	0.0120*
(PSI-116.667)*(Temp-98.3333)	-0.0001	4.07e-5	-2.45	0.0442*
(Temp-98.3333)*(Temp-98.3333)	-0.000294	0.000236	-1.25	0.2519
(Temp-98.3333)*(LogTime-6.08605)	-0.002137	0.003467	-0.62	0.5571



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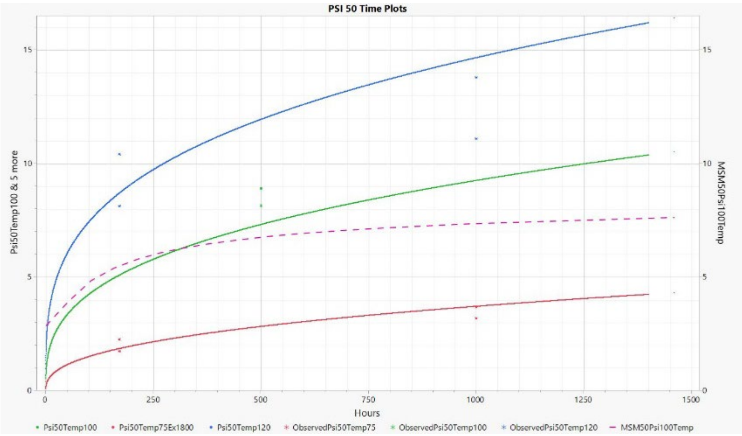
Analysis

- The extent of the quadratic effect of PSI was unexpected. There was less observed Creep in the 200 PSI samples than for 100 PSI
  - This aided in identifying a new mechanism affecting the Creep: under exceptionally high pressure, the pad would flatten and spread the pressure over a wider contact area. This caused the protective coating to experience less strain at the highest applied pressure than at the middle pressure of 100 PSI.

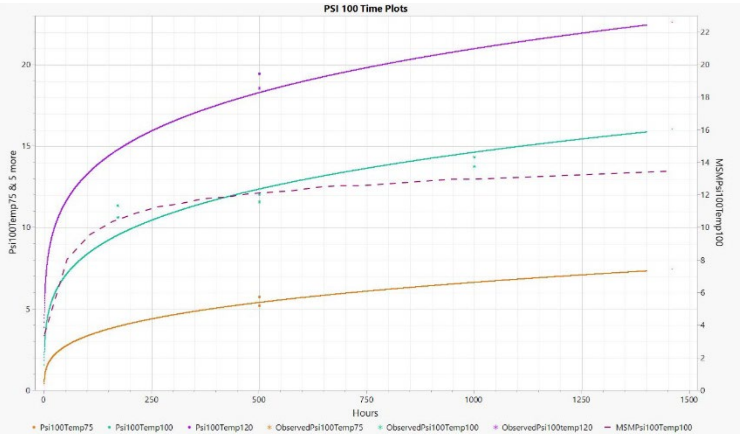
Model Predictions & Comparison to Materials Model

- Model was used to predict how the depth of striations changes over time under different conditions
- Additionally, the model helped validate the materials model which was built from test case 1 data

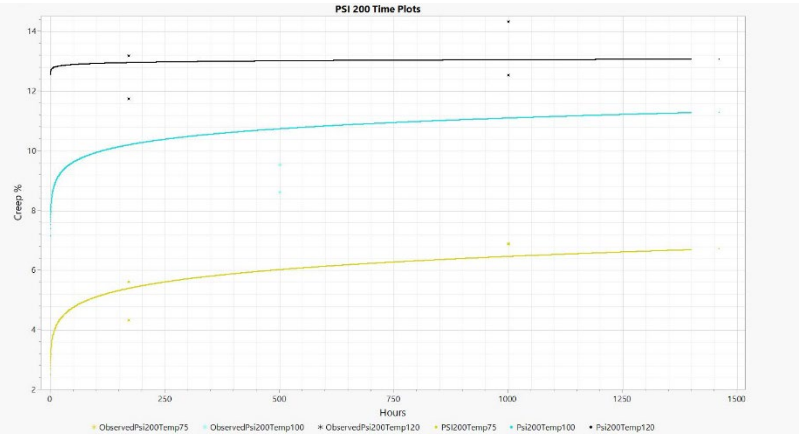
50 psi



100 psi



200 psi



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Analysis

- The model was able to detect effects on Creep with a high degree of significance, and to typically predict within 0.05 transformed units
  - This translated to a residual of approximately 1 mil (1/1000th of an inch) at lower creep values of 2 mils, to 10 mils of residual at high creep values around 14 mils due to the natural log relationship.
- This was able to satisfy both the goals of general characterization, and of identifying when there would be Creep of practical significance to the program.

Conclusions

- The reduced design was built to the program's prior knowledge of system variability and their identified threshold for an effect of practical significance
- This was able to meet the program's needs with much lower cost of resources
  - The program was able to save approximately $200,000 in testing space and apparatus
  - The program was also able to secure the reduced space in less time, saving schedule impact of the unanticipated testing need

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