Profile Monitoring via Eigenvector Perturbation

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$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

$$H_0 : f(\cdot) = f^0(\cdot)$$
$$H_a : f(\cdot) \neq f^0(\cdot)$$

$$T = 1$$

$$H_0 : \quad f^1(\cdot) = f^0(\cdot)$$
$$H_a : \quad f^1(\cdot) \neq f^0(\cdot)$$

$$T = 2$$

$$\begin{array}{llll} H_0: & f^1(\cdot) = f^0(\cdot) & H_0: & f^2(\cdot) = f^0(\cdot) \\ H_a: & f^1(\cdot) \neq f^0(\cdot) & H_a: & f^2(\cdot) \neq f^0(\cdot) \end{array}$$

$$T = 3$$

$H_0:$	$f^1(\cdot) = f^0(\cdot)$	$H_0:$	$f^2(\cdot) = f^0(\cdot)$	$H_0:$	$f^3(\cdot) = f^0(\cdot)$
$H_a:$	$f^1(\cdot) \neq f^0(\cdot)$	$H_a:$	$f^2(\cdot) \neq f^0(\cdot)$	$H_a:$	$f^3(\cdot) \neq f^0(\cdot)$

$$T = 4, 5, \ldots$$

$H_0:$	$f^1(\cdot) = f^0(\cdot)$	$H_0:$	$f^2(\cdot) = f^0(\cdot)$	$H_0:$	$f^3(\cdot) = f^0(\cdot)$	$\cdots$
$H_a:$	$f^1(\cdot) \neq f^0(\cdot)$	$H_a:$	$f^2(\cdot) \neq f^0(\cdot)$	$H_a:$	$f^3(\cdot) \neq f^0(\cdot)$	$\cdots$

Definition (Profile)

A profile is a noise perturbed functional relationship such that

$$y_i^t = f(\boldsymbol{x}_i^t) + \epsilon_i^t, \quad i \in [n], \quad t = 1, 2, \ldots$$

where $y_i^t \in \mathbb{R}$ is a noisy response and $\boldsymbol{x}_i^t \in \mathbb{R}^d$ are known predictors.

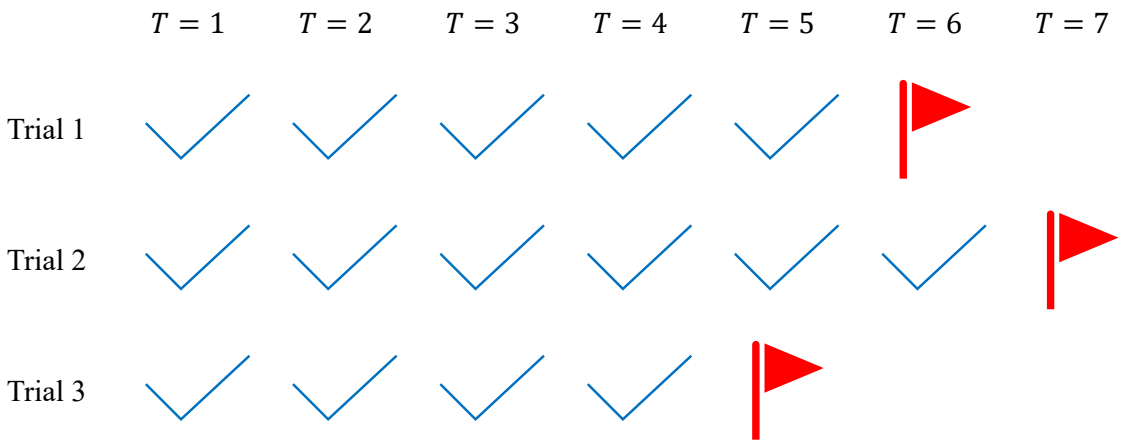
Change Point Framework

For fixed $\tau < T$, for all $i \in [n]$,

$$y_i^t = \begin{cases} f^0(\boldsymbol{x}_i^t) + \epsilon_i^t, & t \leq \tau \\ h(\boldsymbol{x}_i^t) + \epsilon_i^t, & \text{else} \end{cases}$$

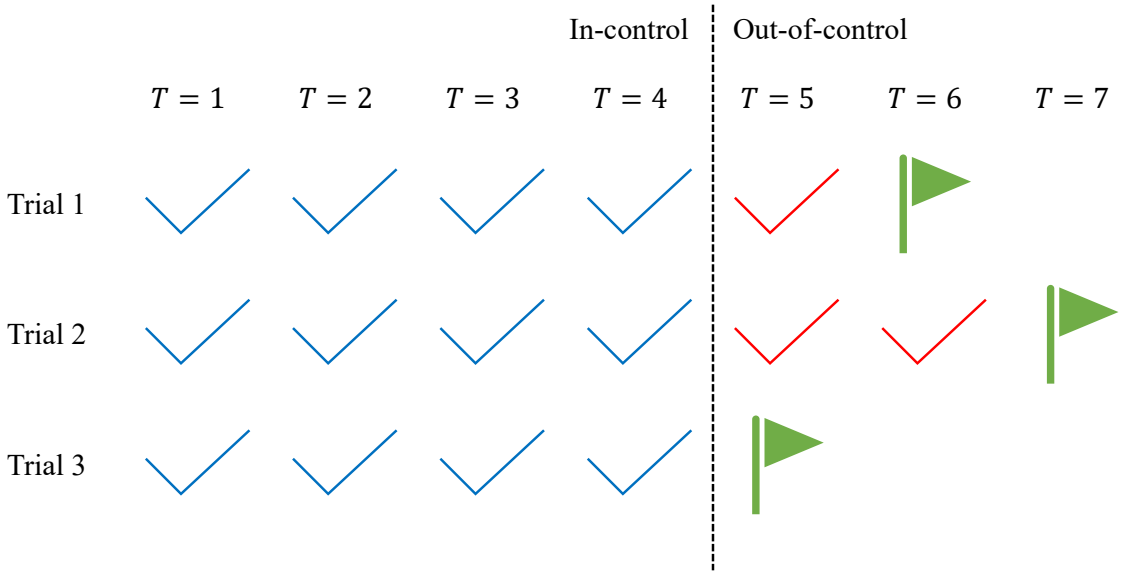
$$\begin{aligned} H_0 : &\quad f^0 = f^1 = \cdots = f^T \\ H_a : &\quad f^0 = f^1 = \cdots = f^\tau \neq f^{\tau+1} = \cdots = f^T \end{aligned}$$

In-control Average Run Length



$$ARL_0 = \frac{6 + 7 + 5}{3} = 6$$

Out-of-control Average Run Length



$$ARL_1 = \frac{2+3+1}{3} = 2$$

Analogous Concept	SPC Terminology	Desired ARL values	Typical Values
Specificity (think H_0)	ARL_0	↑	$200, 370$
Sensitivity (think H_1)	ARL_1	↓	$1 - 10$

Traditional ARL_0 values + Streaming Data

- $ARL_0 = 200$
- New profile observed every 10^{-3} seconds

$\Longrightarrow$ On average a false alarm every 0.2 seconds.

Possible Fix

Calibrate to a large ARL_0:

- $ARL_0 = 10^6$
- New profile observed every 10^{-3} seconds

$\Longrightarrow$ On average a false alarm every ~ 16.7 minutes.

... But doing so typically comes at the cost of an unacceptably large ARL_1.

Goals

Performance

- Computationally fast
- Large ARL_0 (low false alarm rate)
- Small ARL_1 (quickly detect out-of-control)

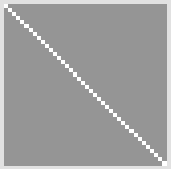
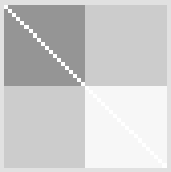
Model Assumptions

- Nonlinear f
- Nonparametric
- Multivariate predictor

Eigenvector Perturbation Control Chart - Main Idea

$w \times w$ Matrix	$\boldsymbol{R}$	$\mathbb{E}[\boldsymbol{R}]$
Leading eigenvector	$\boldsymbol{v}$	$\tilde{\boldsymbol{v}}$

Key idea: $\boldsymbol{v} \approx \tilde{\boldsymbol{v}}$

Scenario	$\mathbb{E}[\boldsymbol{R}]$	$\tilde{\boldsymbol{v}}$	$\|\boldsymbol{v} - \frac{1}{\sqrt{w}}\mathbf{1}\|_2$
All w profiles in-control		$\frac{1}{\sqrt{w}}\mathbf{1}$	small
Mix of in-control and out-of-control profiles		$\propto \xi \begin{bmatrix}\mathbf{1}\\ \mathbf{0}\end{bmatrix} + \begin{bmatrix}\mathbf{0}\\ \mathbf{1}\end{bmatrix}$	large

Declare "out-of-control" if $\|\boldsymbol{v} - \frac{1}{\sqrt{w}}\mathbf{1}\|_2$ is too large.

Speed

Method	Min	Median	Max
Eigenvector Perturbation	0.000	0.001	0.060
Li et al.[2]	0.021	0.025	0.041
Iguchi et al.[1]	15.452	28.003	41.365

Table: Time spent deciding to raise an alarm in seconds

ARL_0 and ARL_1 Performance

Analogous Concept	SPC Terminology	Desired ARL values	Typical Values	Eigenvector Perturbation Control Chart
Specificity (think H_0)	ARL_0	⬆	$200,370$	$> 7 \times 10^6$
Sensitivity (think H_1)	ARL_1	⬇	$1 - 10$	1 $(*)$

Performance Summary

Performance

- ✓ Computationally fast
- ✓ Large ARL_0 (low false alarm rate)
- ✓ Small ARL_1 (quickly detect out-of-control)

Model Assumptions

- ✓ Nonlinear f
- ✓ Nonparametric
- ✓ Multivariate predictor
- ✗ Predictors can change during monitoring

Future Work: How to fix this problem?

Performance

- ✓ Computationally fast
- ✓ Large ARL_0 (low false alarm rate)
- ✓ Small ARL_1 (quickly detect out-of-control)

Model Assumptions

- ✓ Nonlinear f
- ✓ Nonparametric
- ✓ Multivariate predictor
- ✗ Predictors can change during monitoring

Looking forward to seeing you at the poster session!

T. Iguchi, A. F. Barrientos, E. Chicken, and D. Sinha.
Nonlinear profile monitoring with single index models.
Quality and Reliability Engineering International, 37(7):3004–3017, 2021.

C. I. Li, J. N. Pan, and C. H. Liao.
Monitoring nonlinear profile data using support vector regression method.
Quality and Reliability Engineering International, 35(1):127–135, 2019.