Application of Design of Experiments to a Calibration of the National Transonic Facility

Matthew Rhode
NASA Langley Research Center, Hampton, VA

Matthew Bailey
Jacobs Technology Inc., Hampton, VA

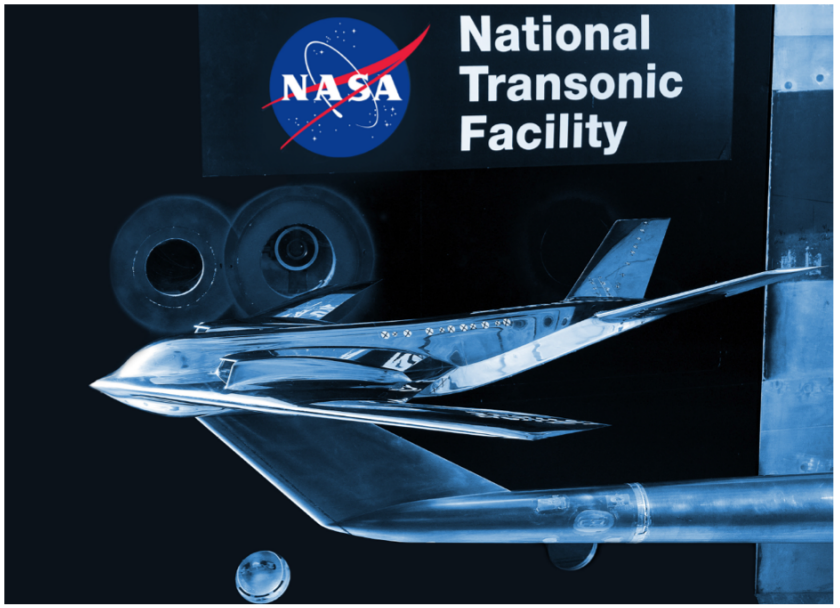
DataWorks 2018, Springfield, VA
20-22 March 2018

Outline

- Description of the National Transonic Facility (NTF)
- Objectives of calibration experiment
- Experiment design, evaluation, and execution
- Regression model and residuals
- Validation of the regression model
- Summary & Future Work

Test Facility

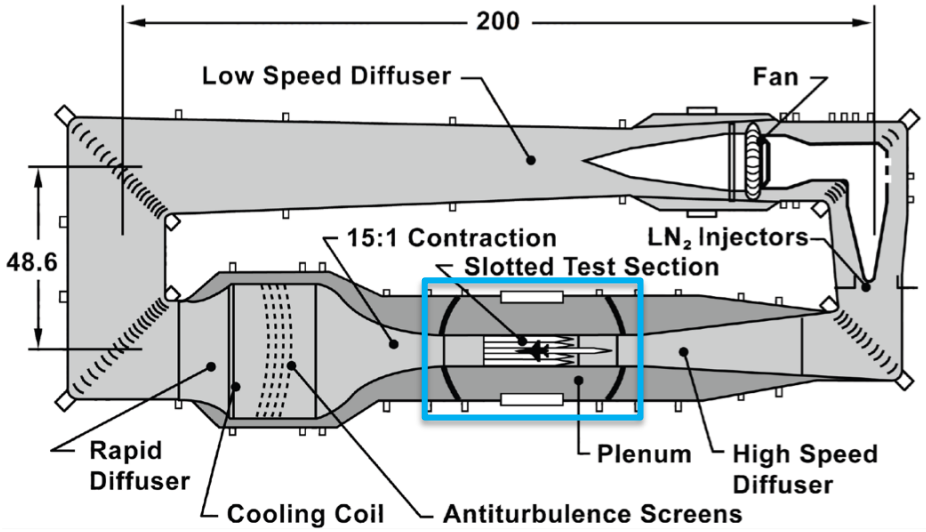
National Transonic Facility

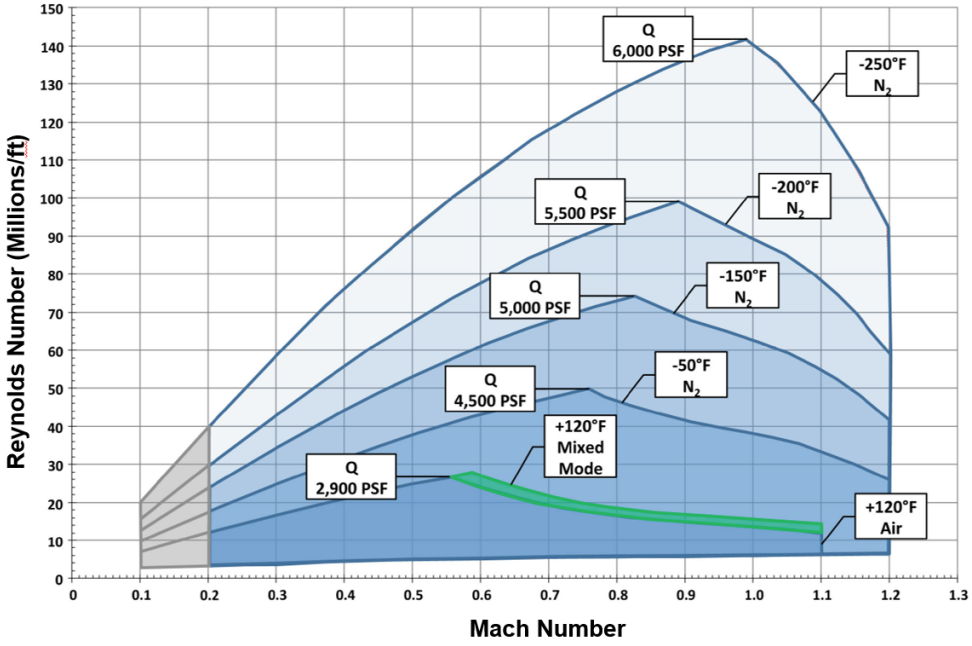


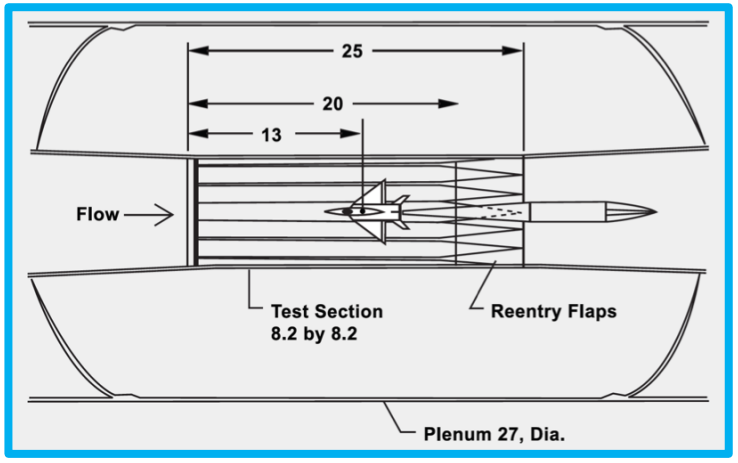
- The world's highest Reynolds number wind tunnel test facility
- Capable of duplicating real flight aerodynamics
- High-pressure, cryogenic, closed circuit, continuous-flow wind tunnel
- Supports advanced aerodynamic concept development and assessment

- Independent pressure and temperature control isolates Reynolds number and dynamic pressure effects
- Operation in air, nitrogen, and mixed modes
- Introduction of new second throat mode of operation

NTF Specifications

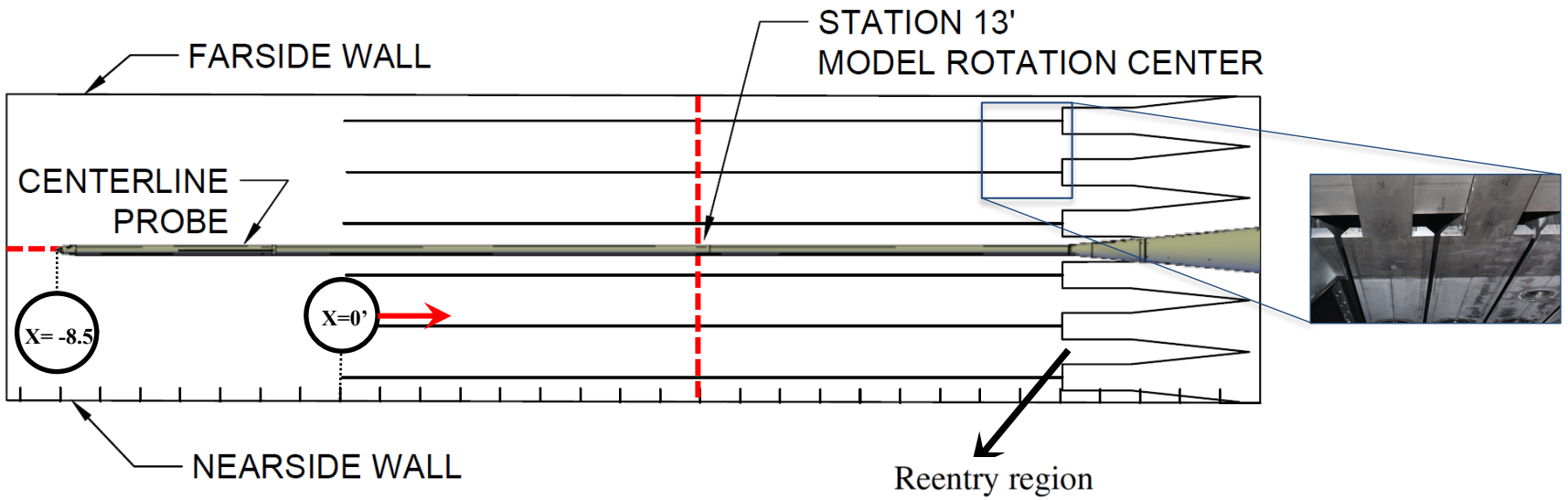




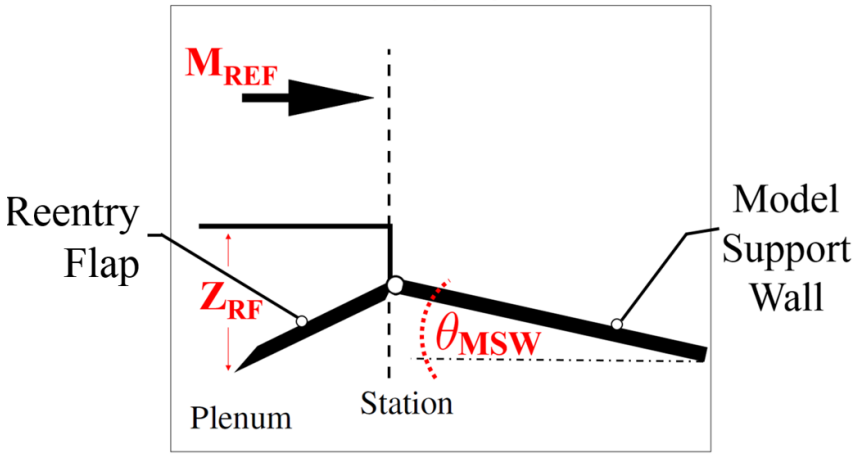


Test Section Size: 8.2 x 8.2 x 25 ft long
Drive Power: 101 MW (135,000 HP)
Mach Range: 0.2 - 1.2
Pressure Range: 15 – 120 psia
Temp Range: -250 to 130 deg F
Reynolds # Range: 0 to 140 x 10^6 per foot

NTF Test Section



TEST SECTION
CEILING PLAN VIEW



Motivation for New Calibration

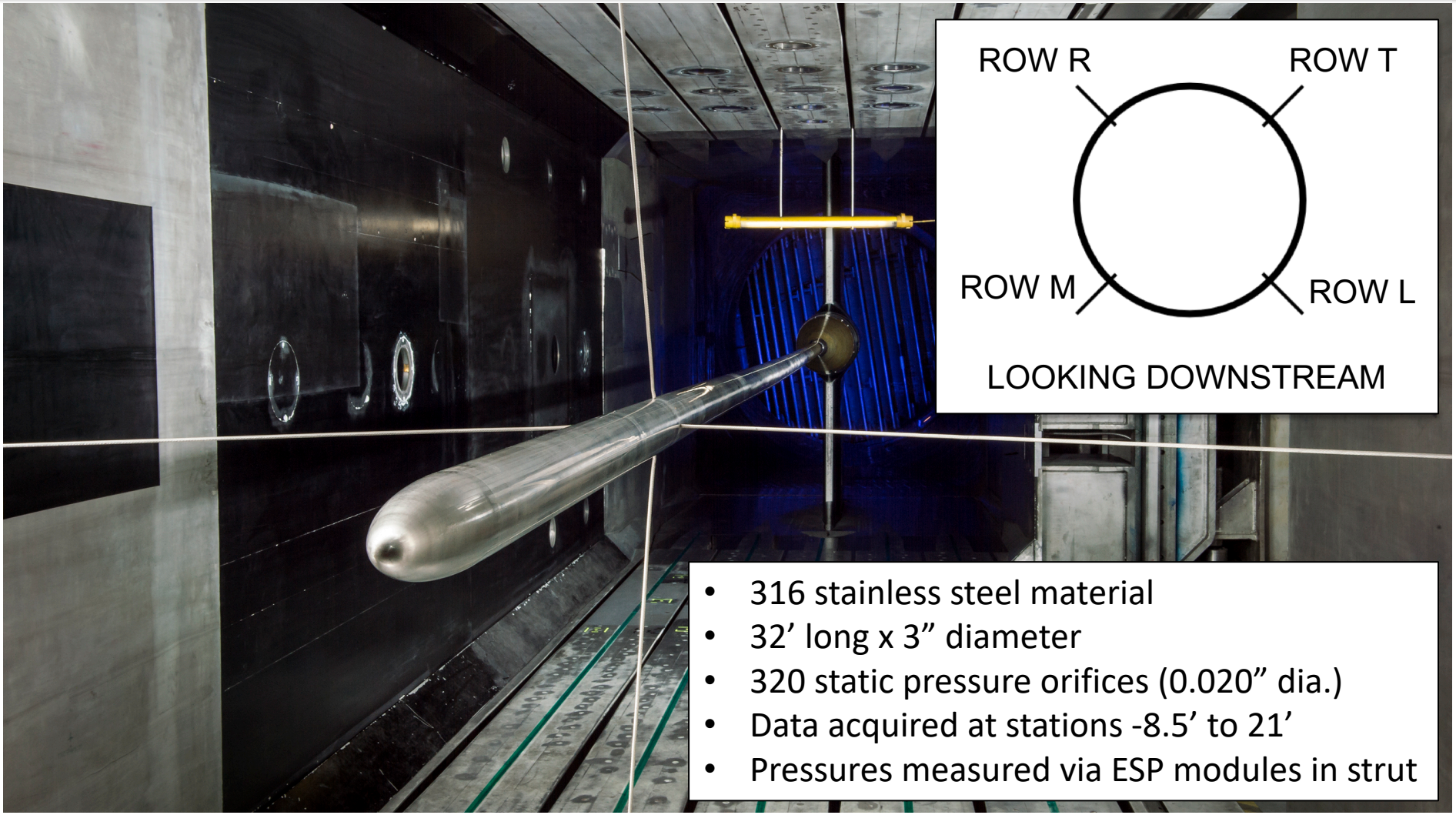
- Recent experimental work has shown that freestream static pressure fluctuations could be significantly reduced by using the moveable Model Support Walls (MSW) and Reentry Flaps (RF) downstream of the test section to choke the flow and create a weak normal shock wave to reduce the upstream propagation of diffuser noise into the test section.
  - Reduction in freestream noise was demonstrated to improve Mach number control and reduce data variability.
- Previous experiments had shown that MSW and RF movements would also change the mean flowfield in the test section.
- Thus, a new calibration would be required to characterize the MSW and RF effects.

Objective of Calibration Experiment

- Characterize effects of Model Support Wall (MSW) and Reentry Flap (RF) movements on centerline static pressure distribution over a range of total pressure and freestream Mach number at four total temperature (T_T) conditions
  - Air: T_T = 120 °F
  - N_2: T_T = -50, -150, -250 °F
- Develop continuous regression models of the local centerline Mach number in the test section ($10 \leq X \leq 16$) as a function of:
  - Reference Mach number (MREF)
  - Total pressure (PTPSI)
  - Average model support wall angle (MSWAVG)
  - Average reentry flap gap height (RFGAPAVG)
  - Location along the centerline (X_PIPE)
- Desired confidence interval (CI) on local Mach number from $10 \leq X \leq 16$: ±0.00025

Instrumentation & Measurements

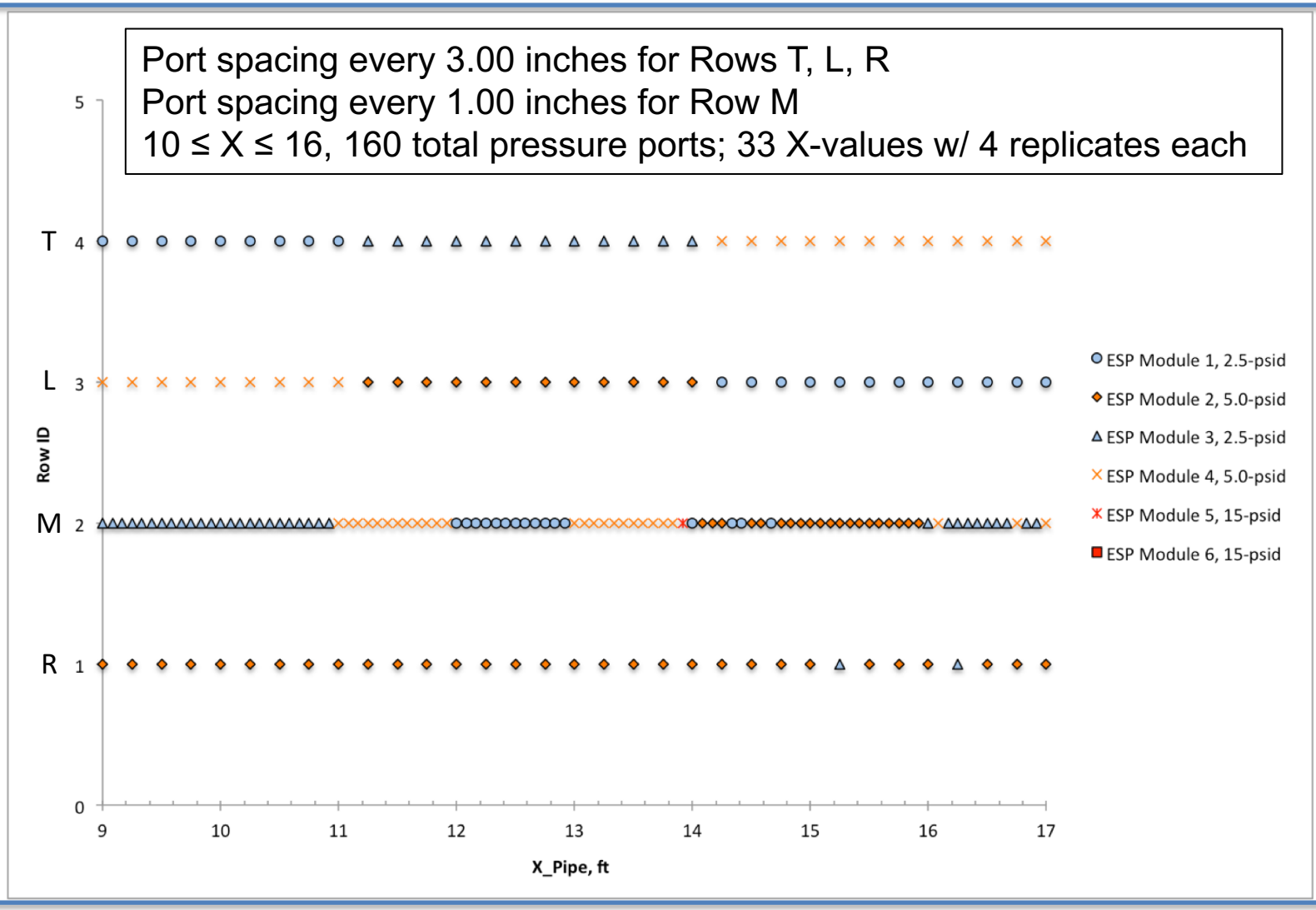
Centerline Static Pipe in Test Section



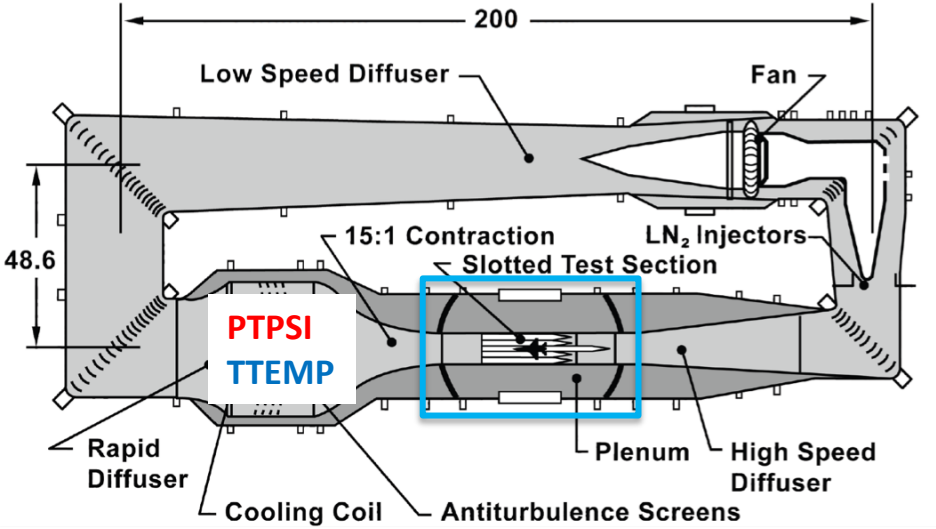
- 316 stainless steel material
- 32’ long x 3” diameter
- 320 static pressure orifices (0.020” dia.)
- Data acquired at stations -8.5’ to 21’
- Pressures measured via ESP modules in strut

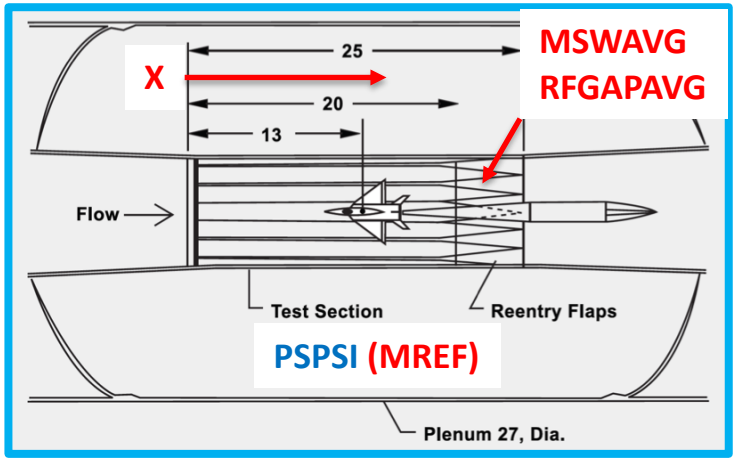
Port Locations in Test Section

Port spacing every 3.00 inches for Rows T, L, R
Port spacing every 1.00 inches for Row M
10 ≤ X ≤ 16, 160 total pressure ports; 33 X-values w/ 4 replicates each



Test Section Measurements





$$\frac{P_T}{P_S} = \left(1 + \frac{\gamma - 1}{2} M_{ref}^2\right)^{\gamma/(\gamma-1)}$$

$$M_{ref} = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_T}{P_S}\right)^{(\gamma-1)/\gamma} - 1\right]}$$

(Factor)

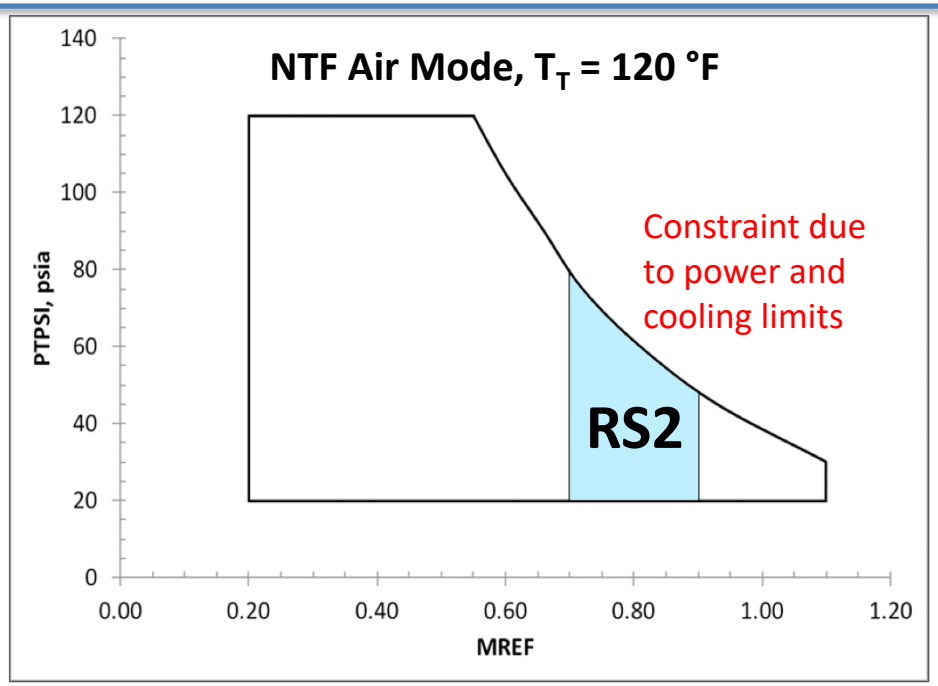
Similarly…

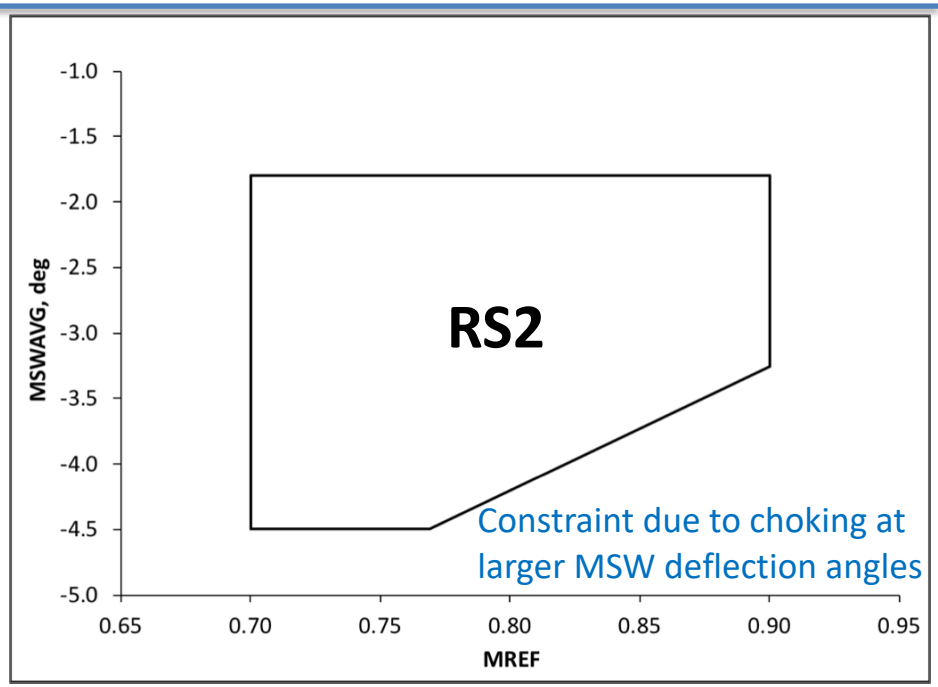
$$M_{pipe} = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_T}{P_{pipe}}\right)^{(\gamma-1)/\gamma} - 1\right]}$$

(Response)

Experiment Design

Experimental Factor Ranges





Factor Ranges for RS2

Factor	Range
PTPSI, psia	[20,80]
MREF	[0.7,0.9]
MSWAVG, deg	[-4.5,-1.8]
RFGAPAVG, inches	[-7,-3]
X_PIPE, ft	[10,16]

Constraints on factor settings:
PTPSI vs MREF
MSWAVG vs MREF

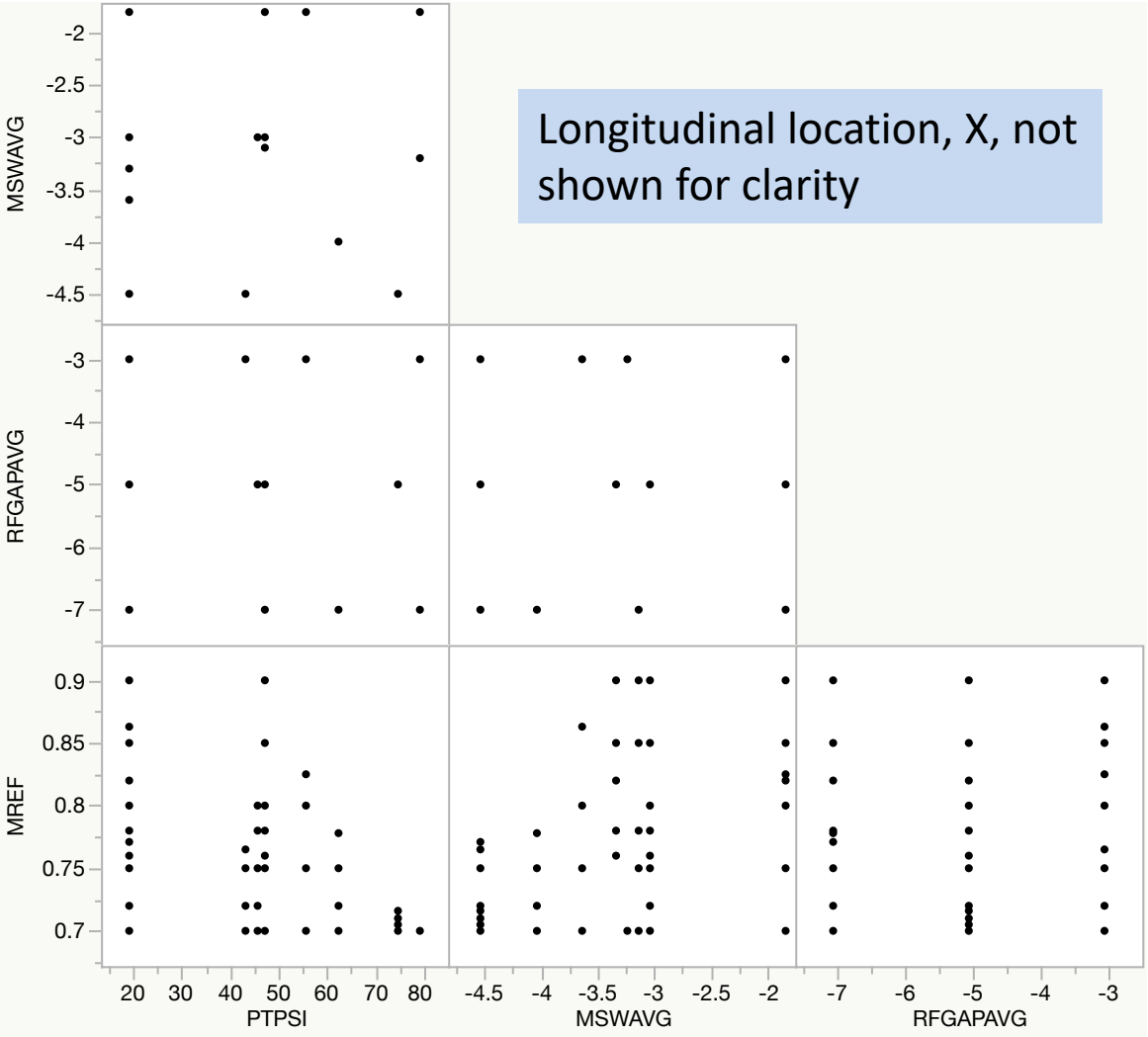
Setting Factors During Experiment

- The time required to set the various experimental factors influenced the design of the experiment
- The MSW and RF were located on the top and bottom walls, and were separated into port and starboard sections
  - Total of four (4) MSW sections and four (4) RF sections
  - Each MSW and RF section angle set by operator, one at a time, from control room using angle reading from encoder
- Facility Safety Head (FSH) required flow reduction to $M < 0.05$ during MSW and RF movements
  - Avoid potential stress on actuators from movement under load
  - Avoid potential asymmetric flow in test section that could stress hardware
- Flow reduced to $M < 0.05$ or dropped altogether during pressure changes to save power costs
- In situ calibration of ESP modules for $\Delta P_T > 20$ psia

Experiment Design

- JMP 12 software, Custom Design tool, I-Optimal design
  - Linear constraints on factor settings
  - 200 random starts for optimal split-plot design algorithm
- 45-run split-split-plot design
  - 15 whole plots for very hard-to-change (VHTC) factors: PTPSI, MSWAVG, RFGAPAVG
  - 3 subplots per whole plot for hard-to-change (HTC) factor: MREF
  - Longitudinal location, X_PIPE, is easy-to-change (ETC): 148 pressure measurements at 73 locations
- Augmented with two additional Mach numbers per whole plot
  - Mach number is relatively easy to change, so this was considered a low-cost means of acquiring additional data that could be used in the analysis, if needed
- Added a complete whole plot replicate for a total of 16 whole plots, 5 subplots per whole plot, and 80 total subplots (80 runs)

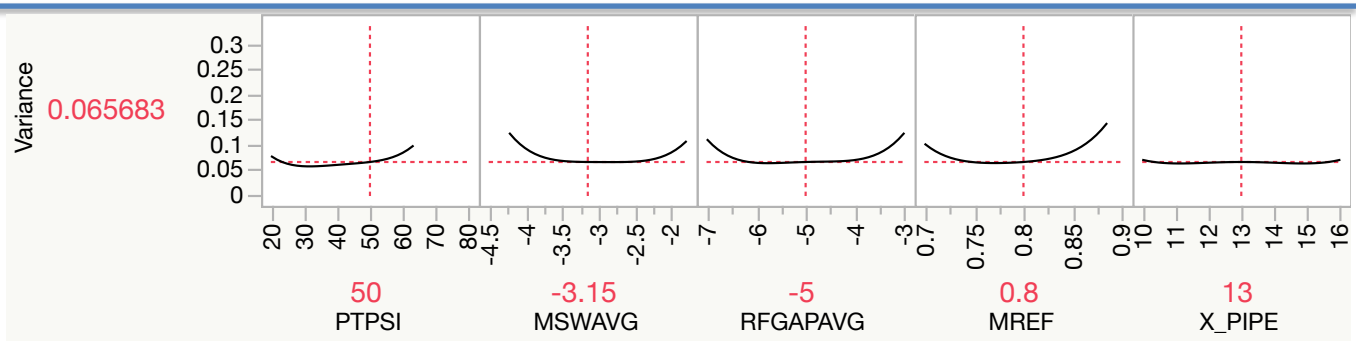
Design Points



Longitudinal location, X, not shown for clarity

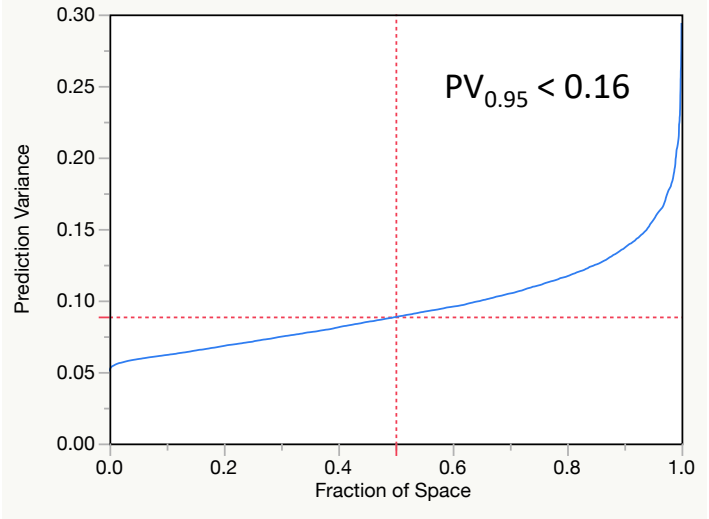
Design Evaluation

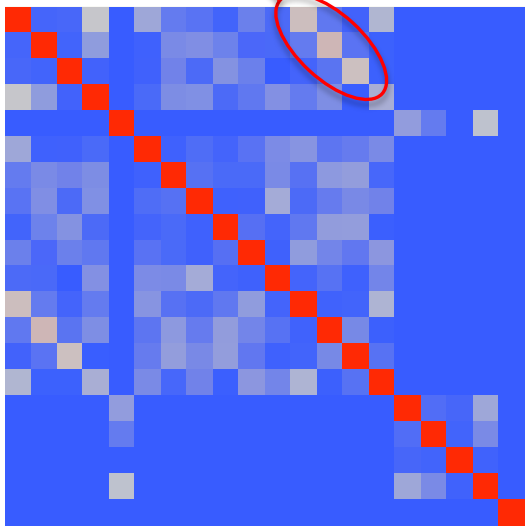
Term	Power
Intercept	0.898
PTPSI	0.842
MSWAVG	0.968
RFGAPAVG	0.997
MREF	0.985
X_PIPE	1
PTPSI*PTPSI	0.76
PTPSI*MSWAVG	0.963
MSWAVG*MSWAVG	0.868
PTPSI*RFGAPAVG	0.986
MSWAVG*RFGAPAVG	0.993
RFGAPAVG*RFGAPAVG	0.917
PTPSI*MREF	0.899
MSWAVG*MREF	0.977
RFGAPAVG*MREF	0.999
MREF*MREF	0.992
PTPSI*X_PIPE	1
MSWAVG*X_PIPE	1
RFGAPAVG*X_PIPE	1
MREF*X_PIPE	1
X_PIPE*X_PIPE	1



$$(\sigma_{wp}/\sigma)^2 = 0.1$$
$$(\sigma_{sp}/\sigma)^2 = 0.1$$
$$\delta/\sigma = 2$$

PTPSI vs PTPSI*MREF: 0.56
MSWAVG vs MSWAVG*MREF: 0.59
RFGAPAVG vs RFGAPAVG*MREF: 0.55



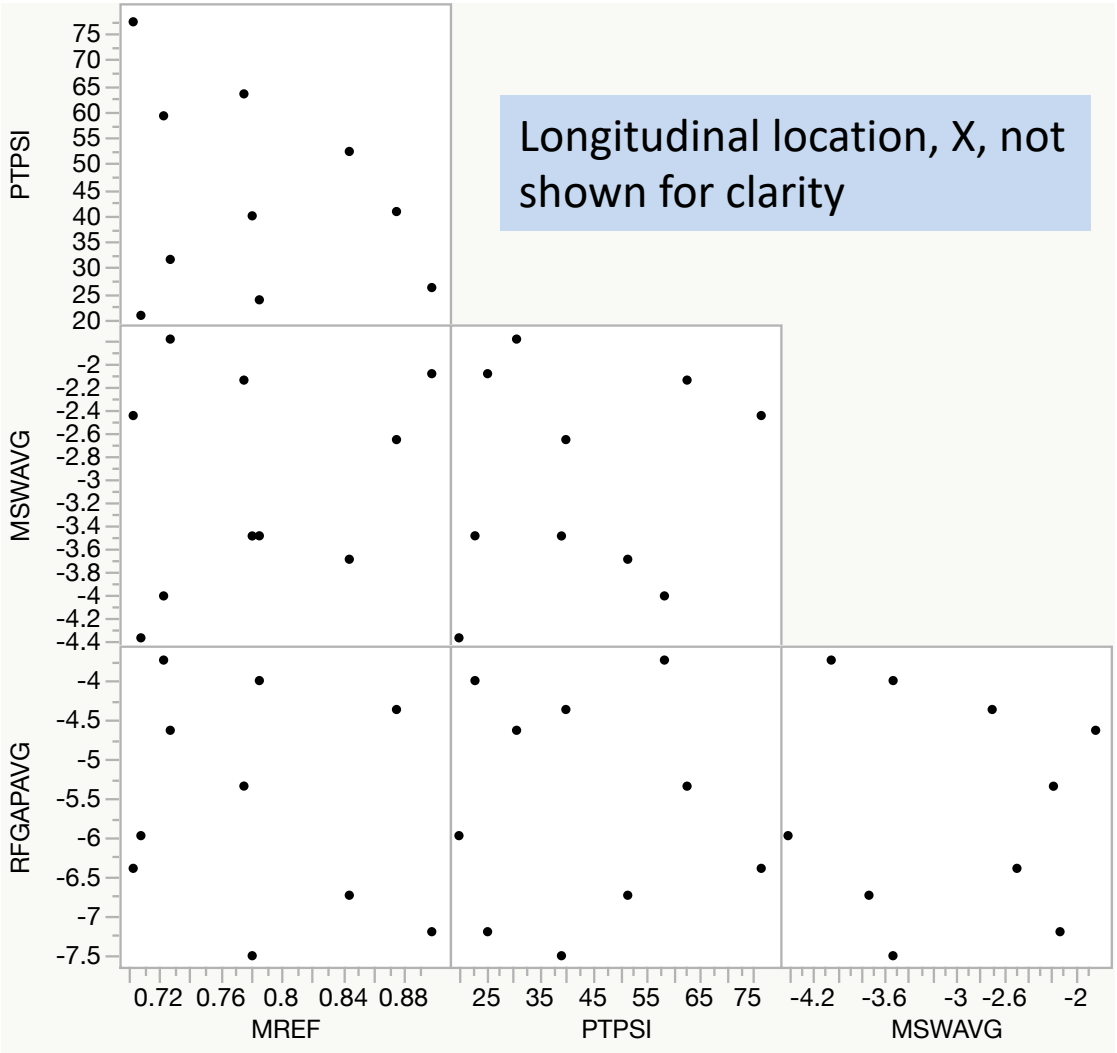


Validation Points

- 12 points were chosen at off-design factor settings to serve as validation points for the regression model
  - Used JMP space-filling design to ensure that there are no large regions of the design space without a validation point

- Validation points were seeded into test matrix between whole plots in quasi-random fashion
  - Assigned according to total pressure (PT) to fall between PT values of adjacent whole plots – catch validation point during pressurization/de-pressurization from one whole plot to the next
  - Done to save time/cost
  - Helped ensure that whole plot factor settings were changed between whole plots

Validation Points in Design Space



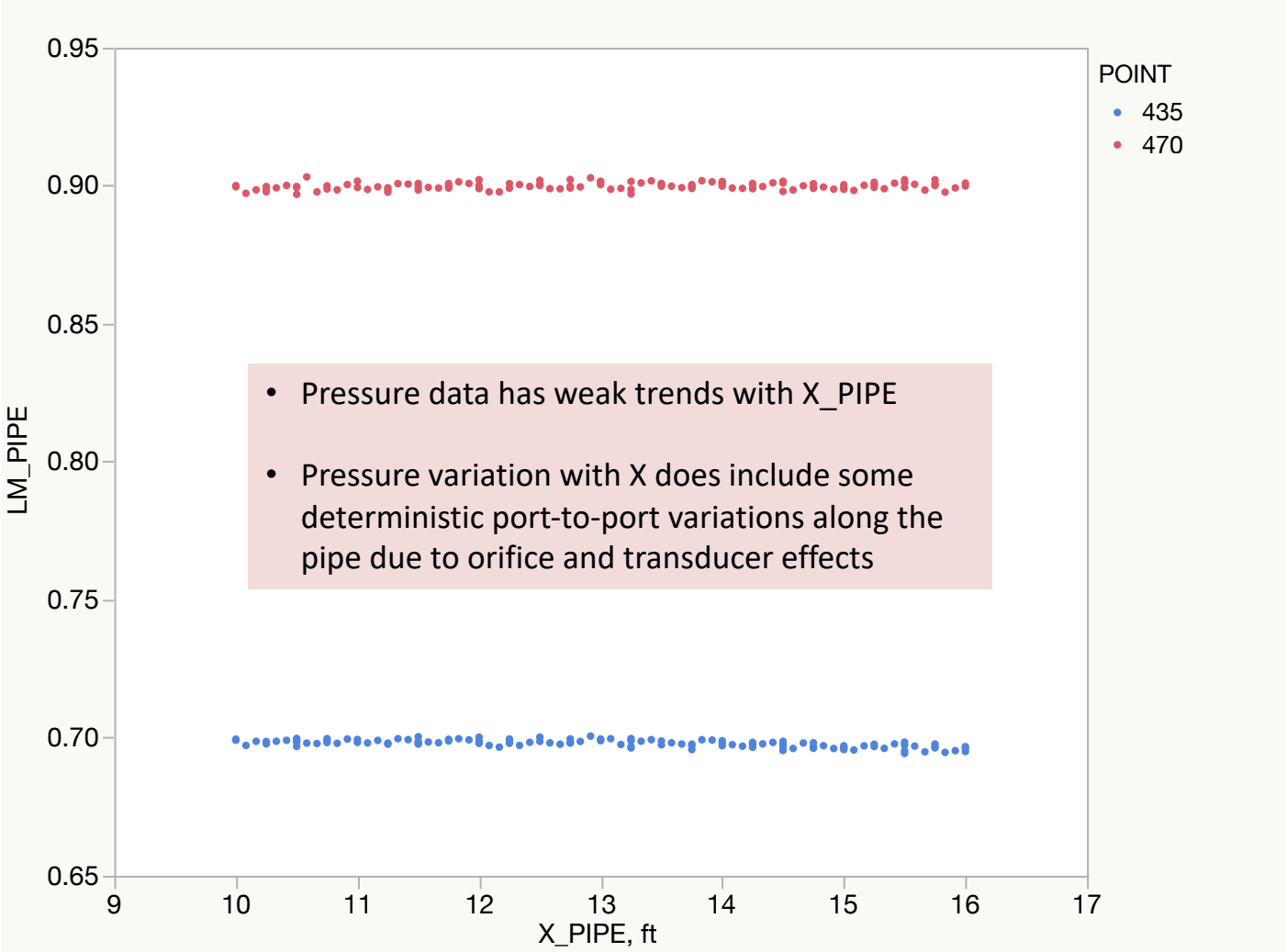
Longitudinal location, X, not shown for clarity

Results

Example Data from Static Pipe

Point 470
MREF = 0.900
PT = 20 psia
MSWAVG = -1.8°
RFGAPAVG = -3 in

Point 435
MREF = 0.700
PT = 44 psia
MSWAVG = -4.5°
RFGAPAVG = -3 in



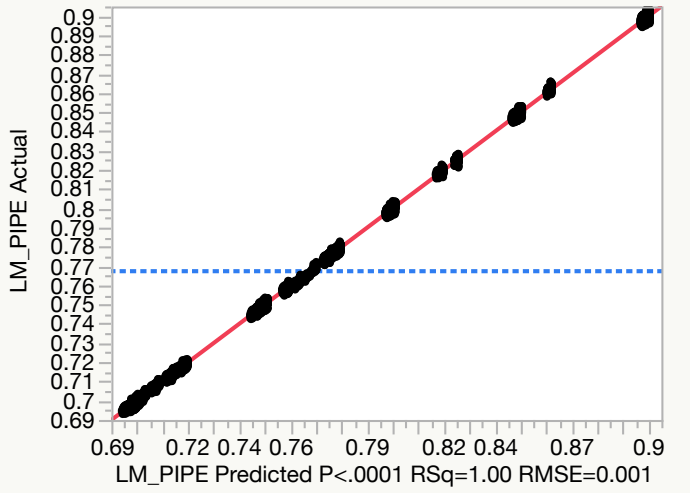
Regression Analysis

- Regression analysis performed using JMP 12 software via Reduced Maximum Likelihood (REML) estimation method
- Started with full 2nd-order model in five (5) factors:
  - VHTC factors: **PTPSI, MSWAVG, RFGAPAVG**
  - HTC factor: **MREF**
  - ETC factor: **X_PIPE**
  - Whole Plot and Subplot considered as Random Effects in regression
  - Response of interest is the local Mach number along the pipe, **LM_PIPE**, as determined from pipe static pressure measurements
  - 21 total terms in regression model (including intercept)
- Used P-values to determine which model terms to eliminate
  - Retain model terms where P-value < 0.05
  - Eliminate model terms one at a time, starting with largest P-value term
  - Reduced 2nd-order model with 13 terms

Regression Results

Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	VIF
Intercept	0.0089756	0.000621	40.54	14.44	.
MREF	0.9946736	0.000526	66.51	1890.0	1.5398179
PTPSI	-3.089e-5	4.154e-6	12.68	-7.44	1.5047845
MSWAVG	0.0007289	8.525e-5	11.77	8.55	1.2333443
RFGAPAVG	0.0002938	5.377e-5	11.52	5.46	1.0839386
X_PIPE	-0.000314	5.491e-6	11435	-57.15	1.0005991
(MREF-0.77225)*(PTPSI-46.1044)	-0.000065	2.776e-5	69.06	-2.34	1.2468059
(MREF-0.77225)*(MSWAVG+3.0176)	0.0011207	0.000503	65.28	2.23	1.2167345
(MREF-0.77225)*(RFGAPAVG+5.61282)	-0.001098	0.000278	64.75	-3.95	1.0867374
(MREF-0.77225)*(X_PIPE-13.0365)	0.0003935	8.75e-5	11435	4.50	1.0632923
(MSWAVG+3.0176)*(X_PIPE-13.0365)	0.0001845	5.693e-6	11435	32.42	1.0671941
(RFGAPAVG+5.61282)*(X_PIPE-13.0365)	8.3778e-5	3.515e-6	11435	23.84	1.0061407
(X_PIPE-13.0365)*(X_PIPE-13.0365)	-0.000144	3.466e-6	11435	-41.46	1.0005991



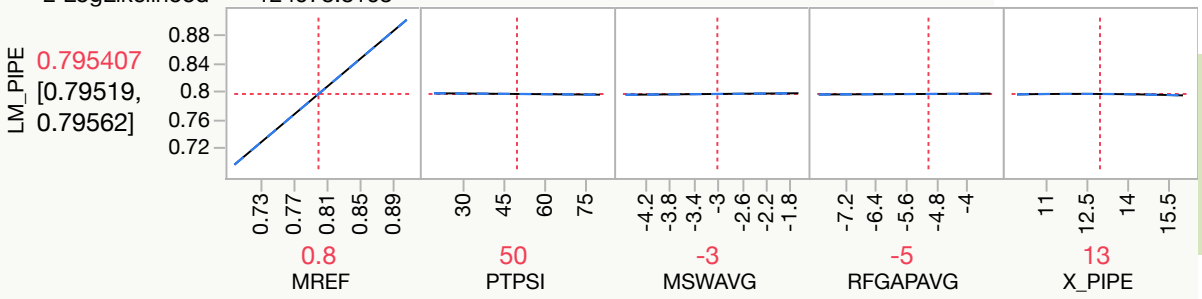
REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Pct of Total
Whole Plots	0.0957013	1.0518e-7	4.7393e-8	1.2289e-8	1.9806e-7	8.544
Subplots	0.02439	2.6805e-8	6.3116e-9	1.4434e-8	3.9175e-8	2.177
Residual		0.0000011	1.4534e-8	1.0711e-6	1.1281e-6	89.278
Total		1.231e-6	4.9642e-8	1.1392e-6	1.3344e-6	100.000

-2 LogLikelihood = -124978.8103

Summary of Fit

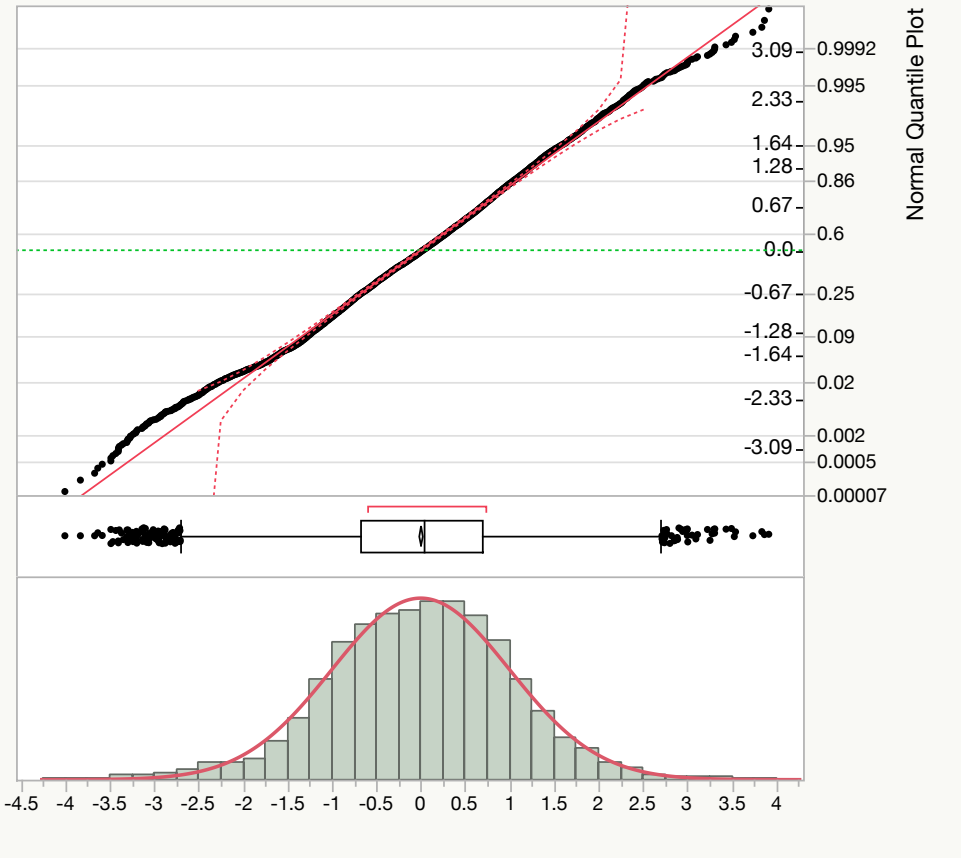
RSquare	0.99974
RSquare Adj	0.99974
Root Mean Square Error	0.001048
Mean of Response	0.767344
Observations (or Sum Wgts)	11520

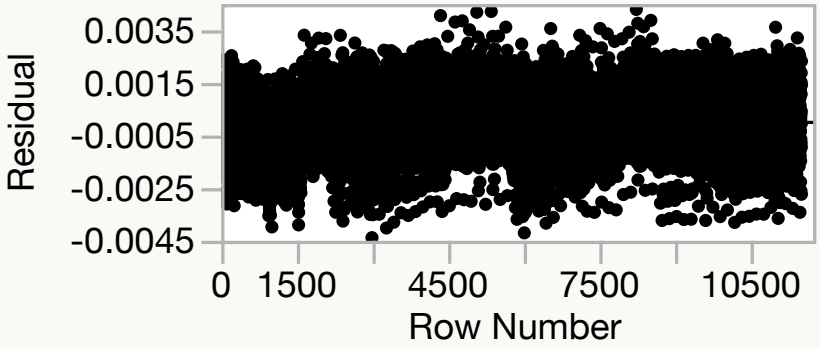


- Regression model coefficient estimates relative to factors in coded units
- Mach gradient, dM/dX, a function of X, MSWAVG, RFGAPAVG, MREF

Regression Model Residuals

Residuals should be NID(0, σ^2)



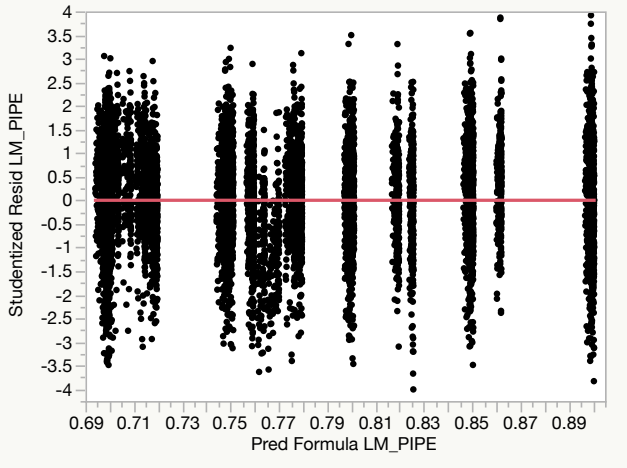


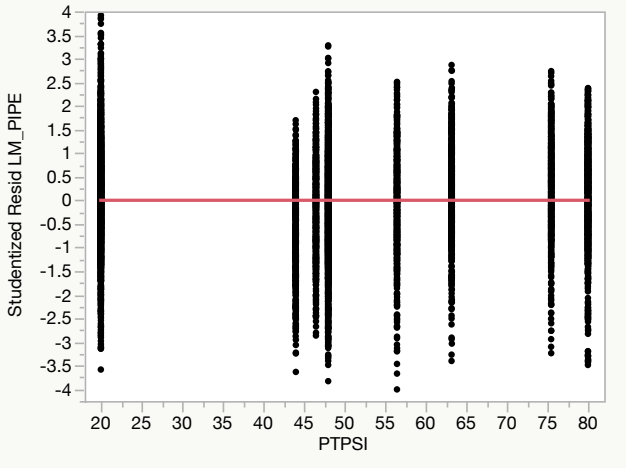
Raw residuals appear to be independently distributed

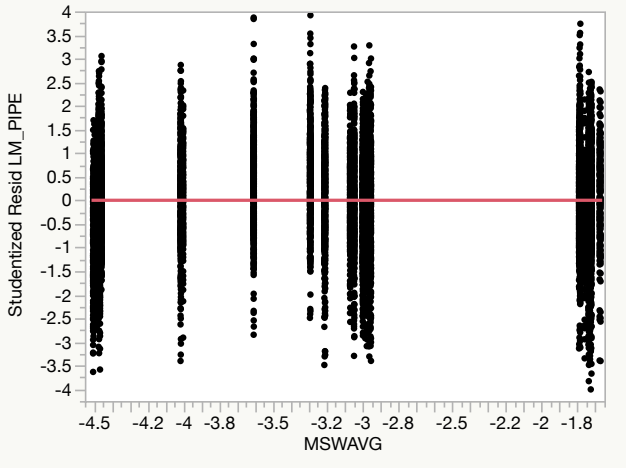
Studentized residuals are normally distributed

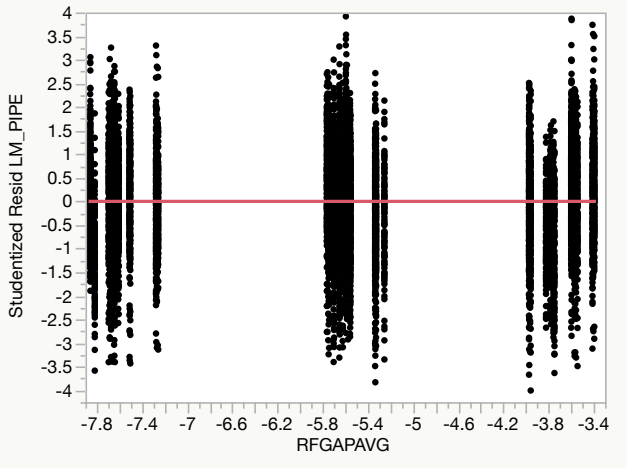
Regression Model Residuals

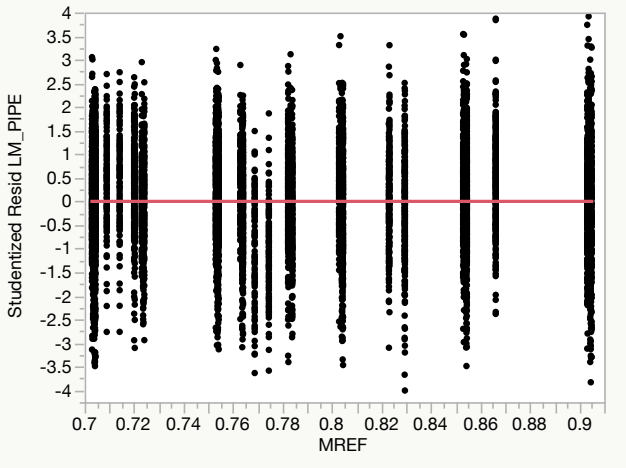
Studentized residuals appear to have a constant variance

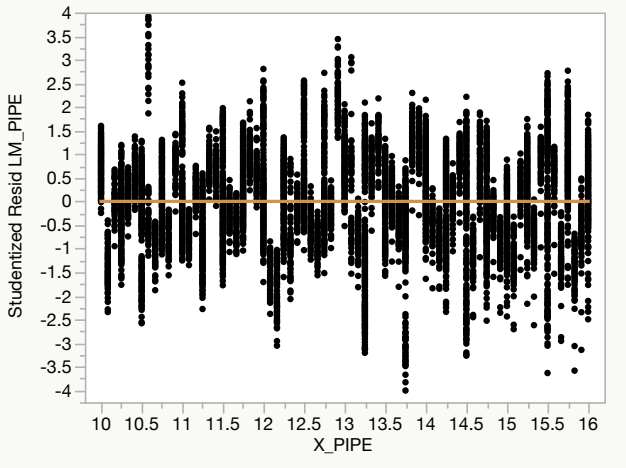




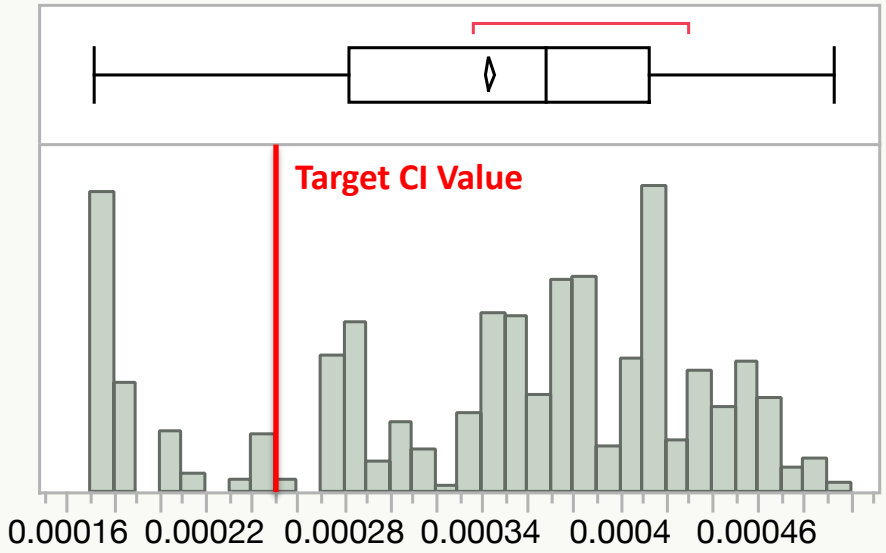








Prediction 95% CI



Quantiles

100.0%	maximum	0.0004928549
99.5%		0.0004859531
97.5%		0.0004681424
90.0%		0.000448044
75.0%	quartile	0.0004129526
50.0%	median	0.0003681475
25.0%	quartile	0.0002818117
10.0%		0.0001799706
2.5%		0.0001723668
0.5%		0.0001711247
0.0%	minimum	0.0001708331

Mean	0.0003431
Std Dev	8.9885e-5
Std Err Mean	8.3746e-7
Upper 95% Mean	0.0003447
Lower 95% Mean	0.0003415
N	11520

Values shown are 2* Standard Error Prediction

Desired 95% CI value is in the lower 25% quartile of the CI values in the design space

Validation Example

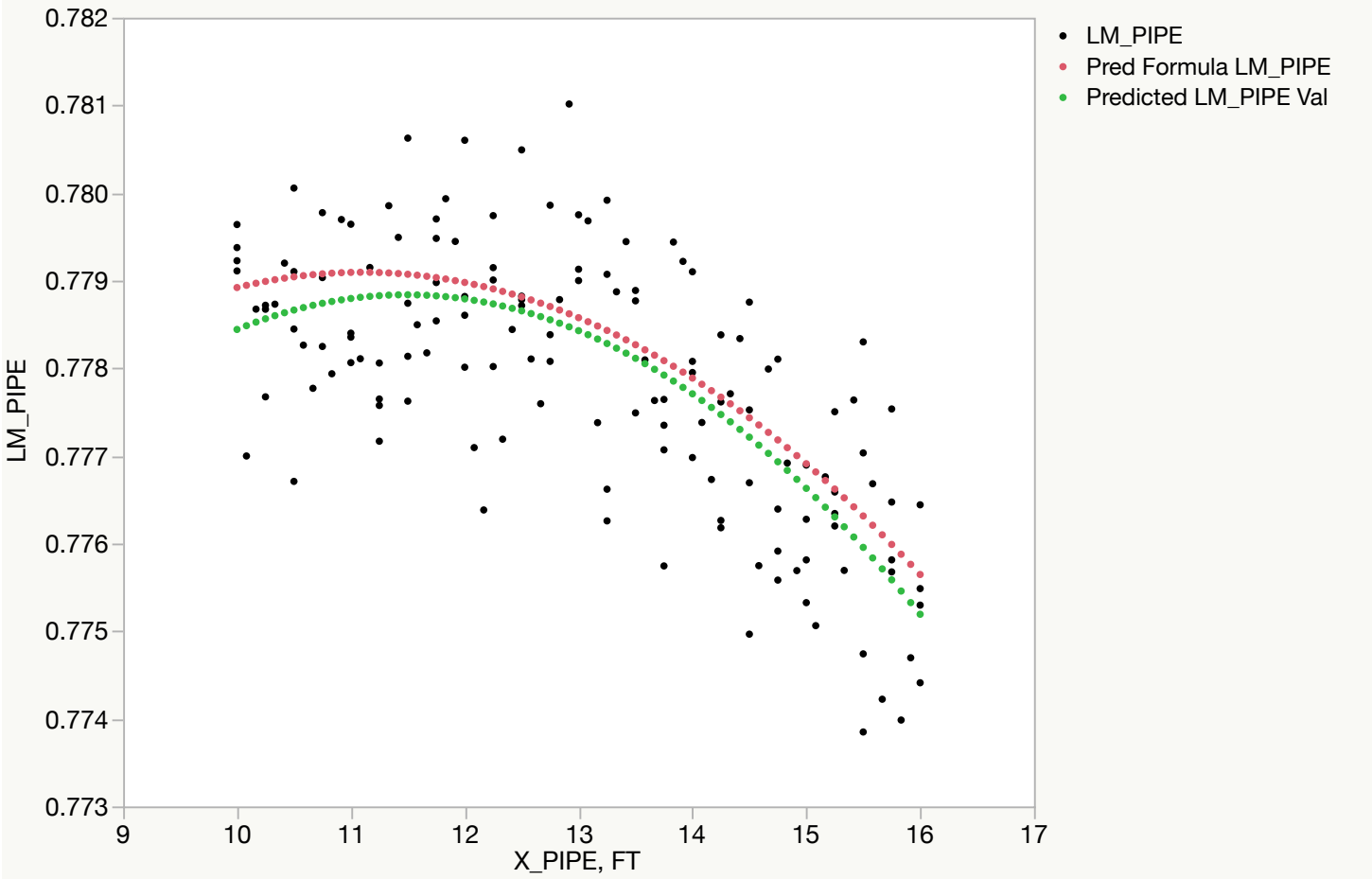
Point 434

PTPSI = 40 psia

MREF = 0.780

MSWAVG = -3.5°

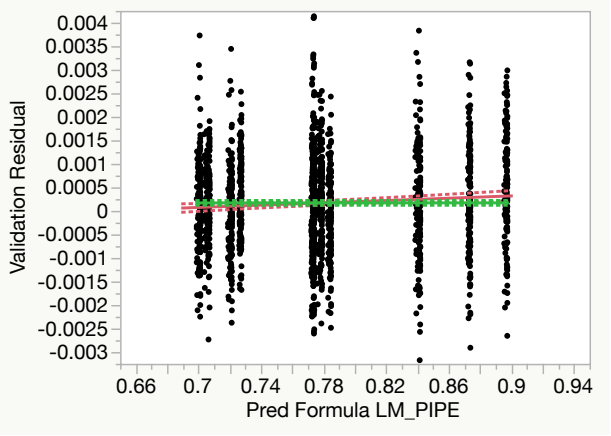
RFGAPAVG = -7.0 in

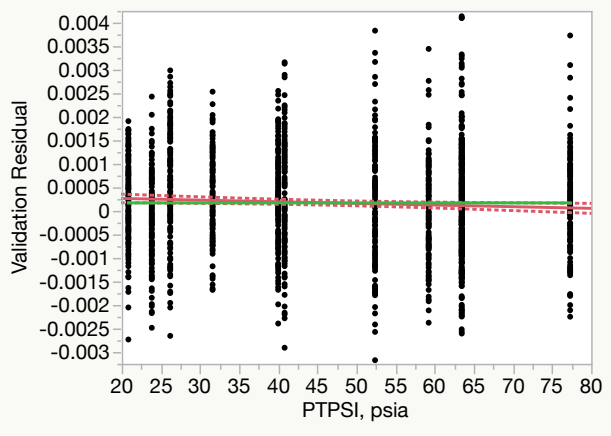


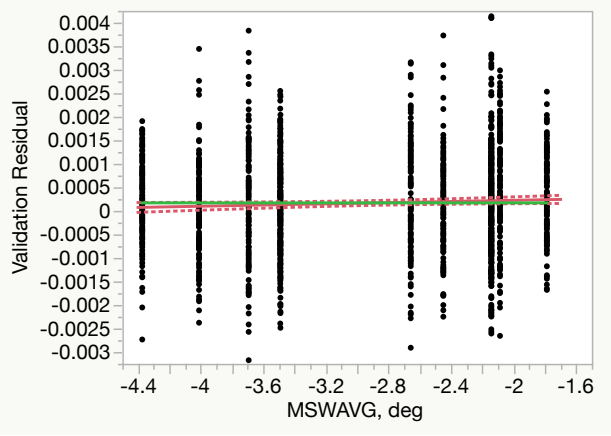
Confidence intervals on regressions not shown for clarity

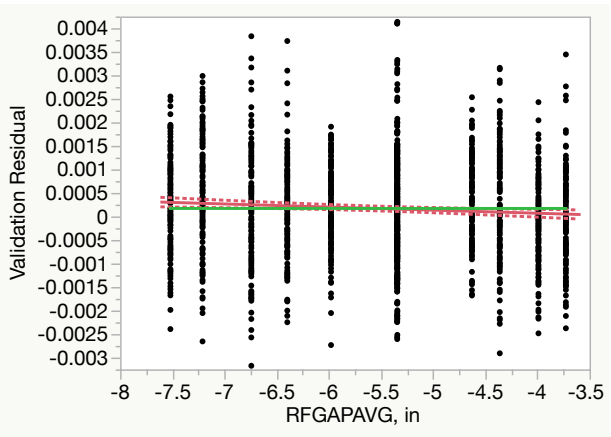
Validation Residuals

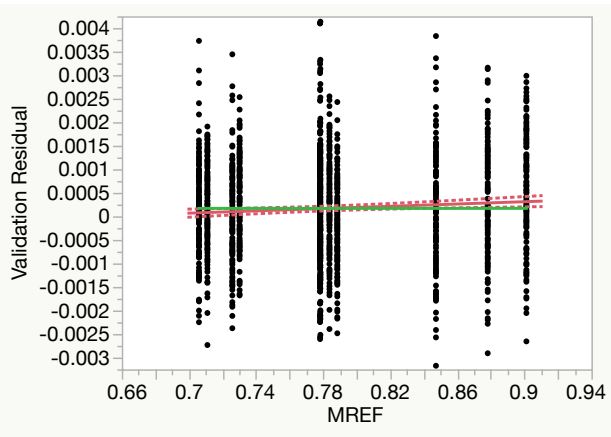
Validation residuals fairly uniform over factor ranges

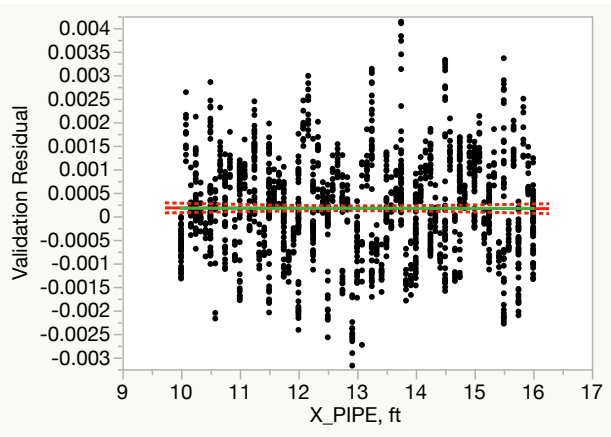










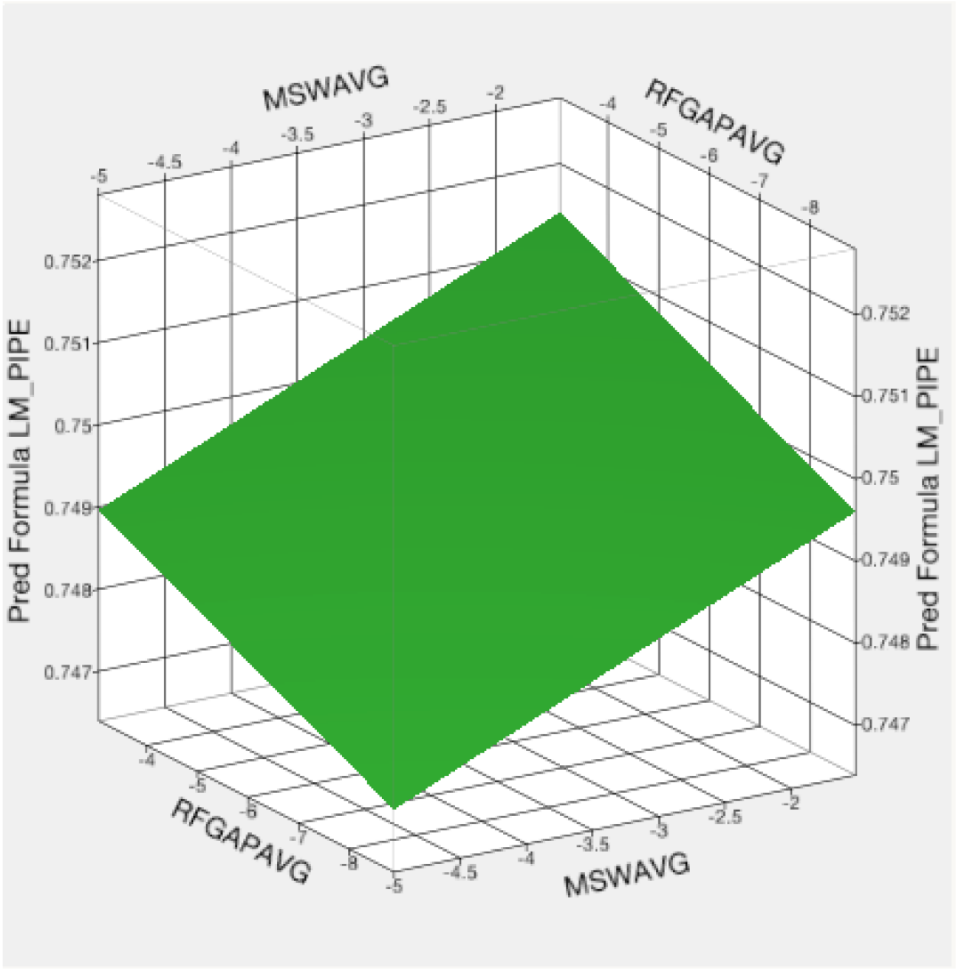


Validation residual mean = 0.00017 ± 0.00005 (95% CI)

Effect of Moveable Walls

Moveable walls have a significant effect on the test section Mach number

An increase in the **magnitudes** of both MSWAVG and RFGAPAVG result in a reduction in test section Mach number of at least an order of magnitude larger than the CI of the prediction



Use of New Regression Model

- $M_\infty = f(M_{REF}, P_T, X, MSW, Z_{RF})$
- Continuous regression function – no lookup tables required
- Solve directly for Mach number at any X station in the test section
- Mach gradient, dM/dX, obtained by taking partial derivative w.r.t. X
  - dM/dX also a continuous function, $g(M_{REF}, X, MSW, Z_{RF})$ – solved directly
  - More accurate functional representation of dM/dX – no longer constant w.r.t. X
  - Use dM/dX along with model cross-sectional area distribution, dA/dX, to obtain buoyancy drag correction using numerical integration
- Allows use of MSW and RF to create second throat downstream of test section to reduce propagation of diffuser noise upstream into test section
  - Improvements in flow stability and flow control
  - Improvements in data repeatability

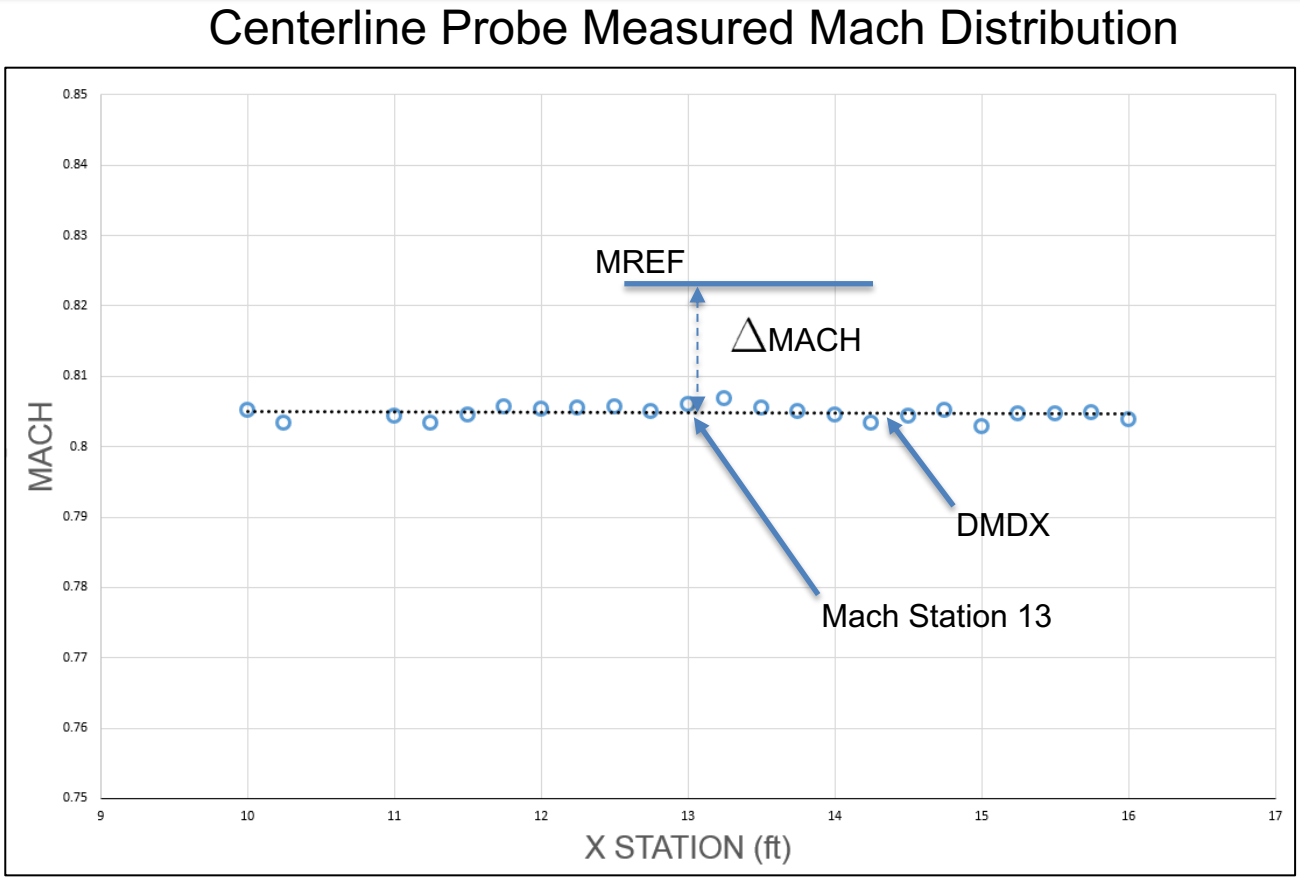
Summary & Future Work

- Calibration experiment performed in NTF using centerline static pipe
  - DOE approach using a split-split-plot design in five factors
  - 16 whole plots, 80 subplots supporting full 2nd-order regression model
  - Experiment showed good resolving power; moderate correlations between main effects and 2-factor interactions
- REML used for regression analysis
  - Yielded reduced 2nd-order model with 13 terms
  - Residuals indicate a well-fitted model
  - Mean 95% CI on prediction is about 36% higher than desired value of 0.00025
- 12 validations runs performed
  - Comparisons with regression model output indicate that the regression model can accurately predict the test section Mach number
  - Weak trends in validation residuals suggest possible room for improvement
- Future work
  - Analysis of calibration experiment data for other total temperature conditions
  - Removal of deterministic pressure orifice/transducer effects along static pipe

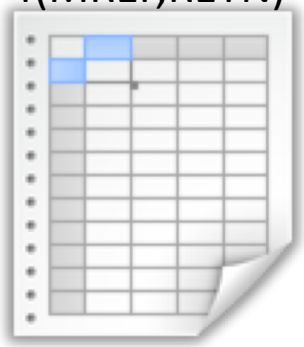
Questions?

Backup Slides

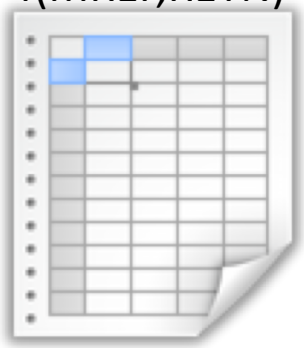
NTF Legacy Calibration Method



DMDX
f(MREF,REYN)



ΔMACH
f(MREF,REYN)



- Four measured pressures averaged at each X-station; compute local Mach number
- Linear regression fit of MACH vs X to obtain slope, DMDX
- ΔMACH is the difference between MREF and the regression value of MACH at Station 13