Spatio-Temporal Modeling of Pandemics

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The Devil’s in the dependency!

Basic Statistical Model

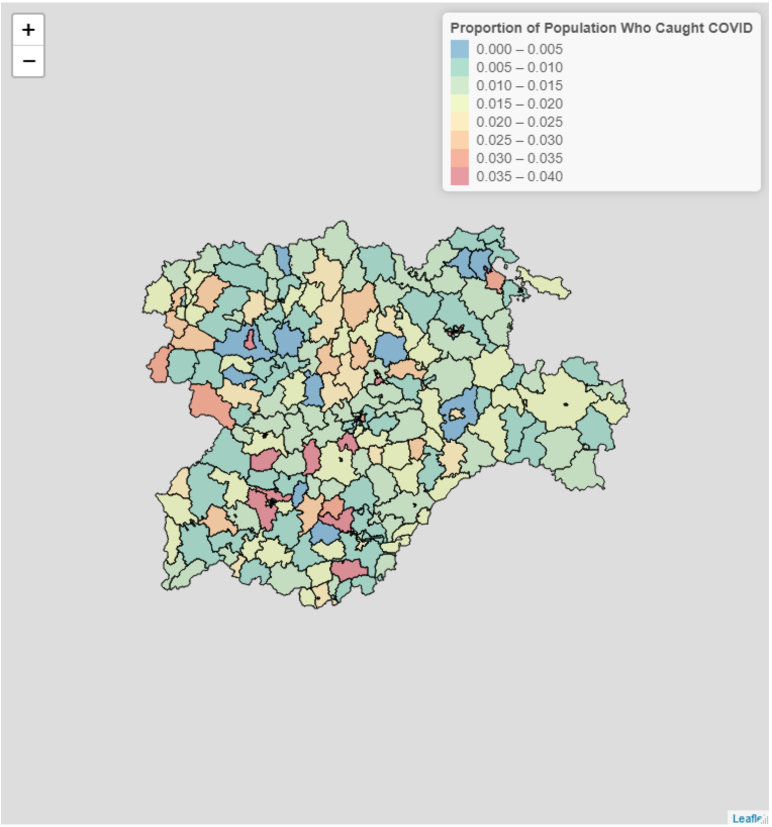
- $\boldsymbol{s_i}$ - Spatial vector, usually location in $\mathbb{R}^2$
- t - time
- $Z(\boldsymbol{s_i}, t)$ - Number of events observed at spatio-temporal location $\boldsymbol{s_i} \times t$

$$Z(\boldsymbol{s_i}, t) \sim Po(\lambda(\boldsymbol{s_i}, t))$$

$$\log(\lambda(\boldsymbol{s_i}, t)) = \beta_0 + \sum_{j=1}^{n} x_{(\boldsymbol{s_i}, t, j)} \beta_j$$

- All spatial-temporal correlation is captured in the large structure covariates

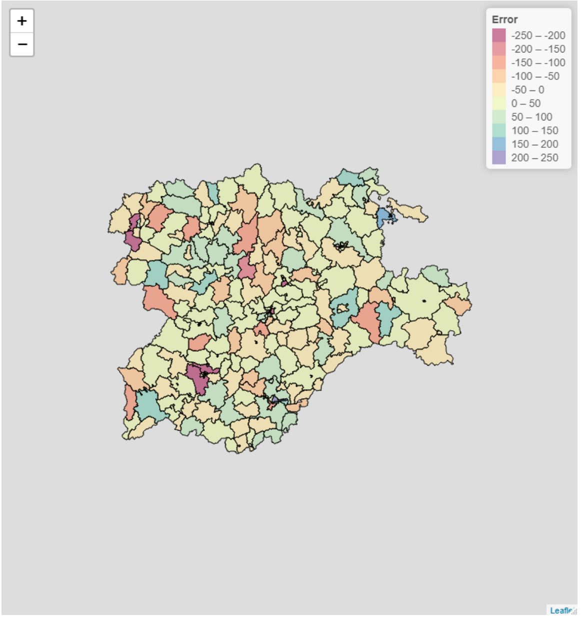
Castilla y Leon Confimred COVID cases



Spatial Only Model with Large Scale Effects

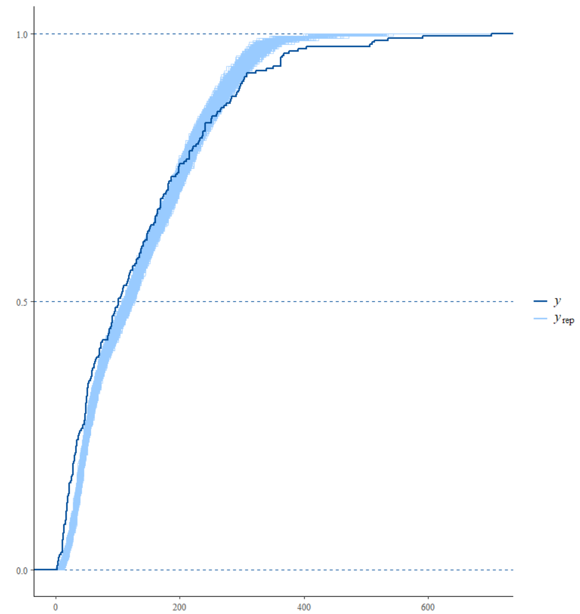
$$Z(\boldsymbol{s_i}) \sim Po(\lambda(\boldsymbol{s_i}))$$
$$\log(\lambda(\boldsymbol{s_i})) = \beta_0 + \log(Pop_{s_i}) + \beta_{Urban} x_{s_i}$$
$$\beta_0, \beta_{Urban} \sim N(0, 10)$$

- $E[\lambda|Z]$ vs Z



Posterior Predictive Checks

- One goal of statistical modeling is to capture key elements of a scientific mechanism in small number of parameters
- Predictive distribution $p(y^{rep}|y) = \int p(y^{rep}|\theta)p(\theta|y)d\theta$
- Posterior predictive checks compare key elements of original data with key elements from generated data



Capturing small scale spatial effects

$$Z(\boldsymbol{s_i}) \sim Po(\lambda(\boldsymbol{s_i}))$$

$$\log(\lambda(\boldsymbol{s_i})) = \beta_0 + \sum_{j=1}^{n} x_{(\boldsymbol{s}_i, j)} \beta_j + \phi(\boldsymbol{s}_i)$$

$$\phi \sim \mathrm{MVN}(\mathbf{0}, \Sigma(\theta))$$

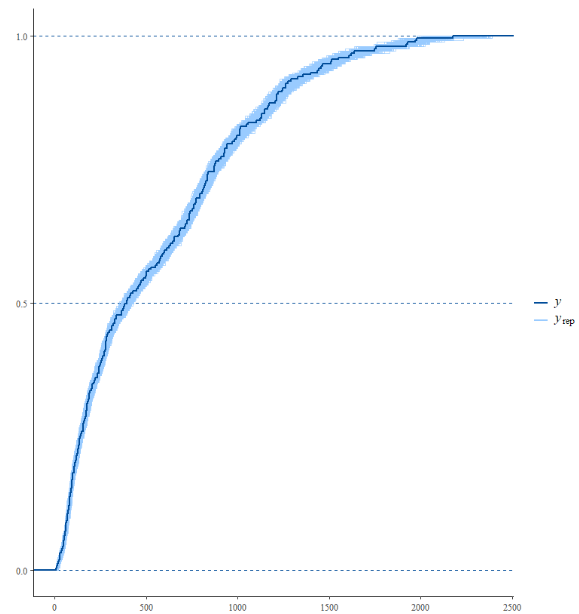
- Structure of $\Sigma(\theta)^{-1}$ yields conditional spatial dependence

$$\phi(\boldsymbol{s}_i)|\phi(\boldsymbol{s}_j) \sim Gau\left(\frac{1}{N|\boldsymbol{s}_i|} \sum_{s_j \in N|s_i|} \phi(\boldsymbol{s}_j), \sigma^2\right)$$

- Precision Matrix only has entries where neighbors exist

Results

- In practice used BYM model (convolves spatial ICAR with heterogeneous RE)
- Even done efficiently 100 times slower, but appears to replicate data well



Alternatively INLA

- Compute marginals of hyper-parameters using Laplace approximation
- Compute condtional distribution of parameters given hyper-parameters and data
- Numerically integrate out hyperparameters
- Efficiently explore parameter space

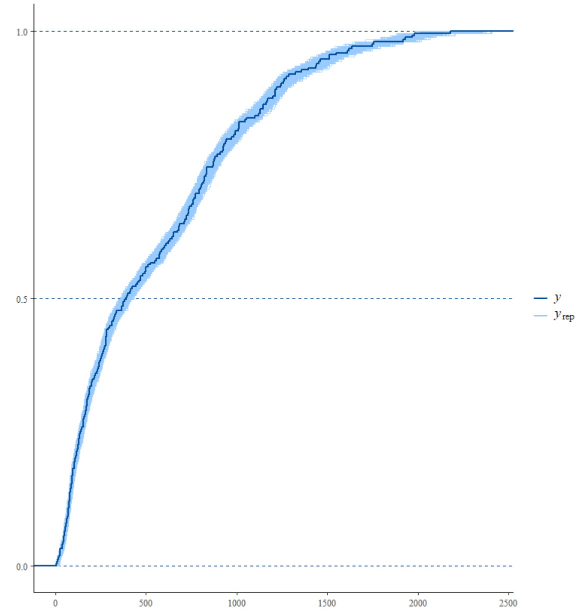
```
inla.formula <- y~ 1+pop+hzone+f(county,model="bym2",
                                 graph = q.mat,constr = TRUE)

model <- inla(inla.formula,family="poisson",data=inla.df,
             control.compute = list(dic=TRUE,cpo = TRUE))
```

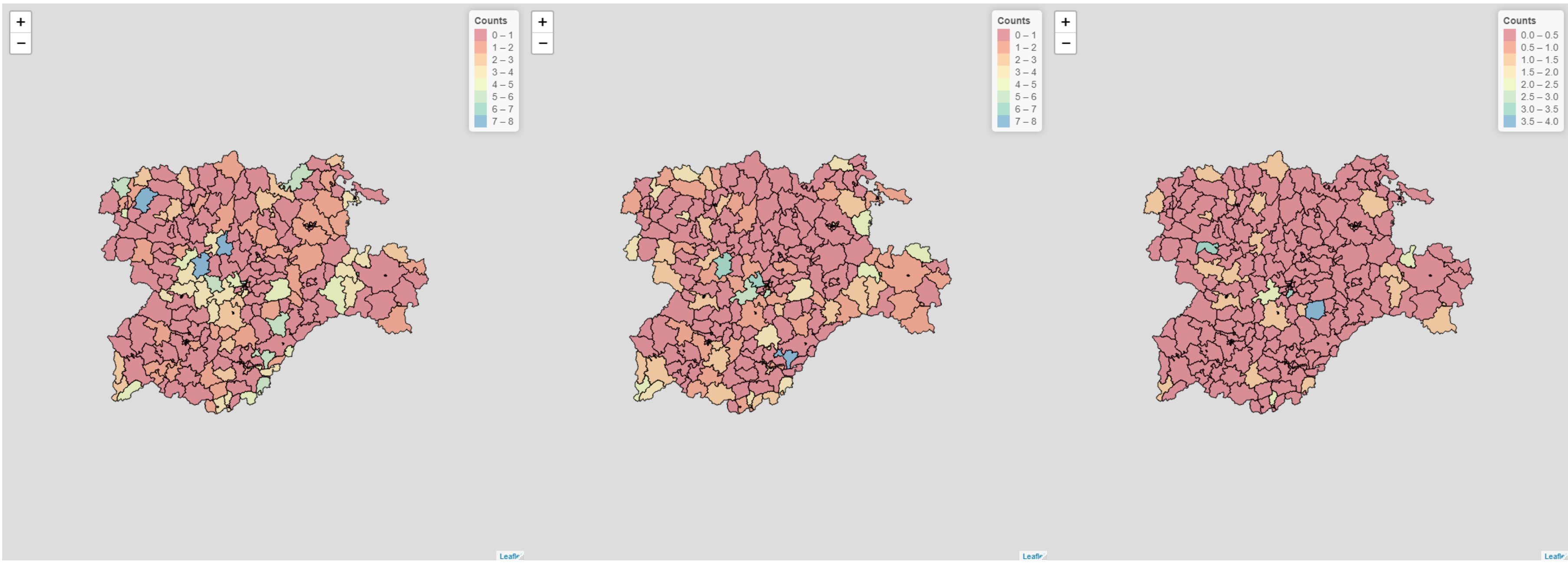
- 12 seconds to run vs 250 seconds per chain in stan

Is this overkill?

- Moran's I fails to reject spatial randomness
- Model with only heterogeneous error



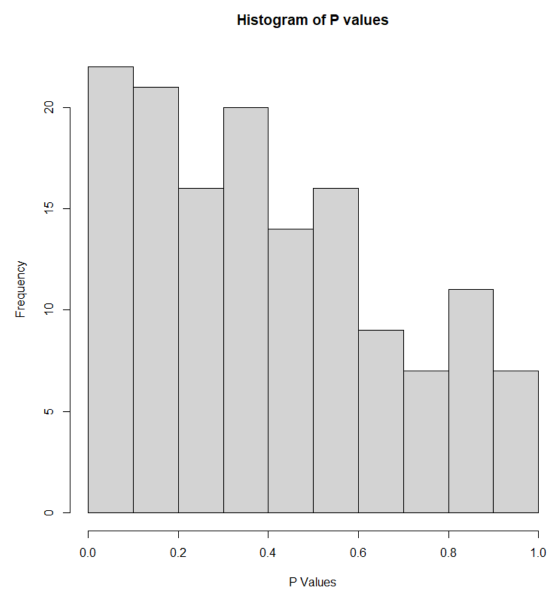
Need to keep in mind dynamic of virus



Hot Spots Emerge and Dissipate

Histogram of Moran's I

- Not Uniformly distributed



Spatial-Temporal Models

- The spatial structure changes as time changes
- Challenge is how to structure model

$$\eta \equiv \log(\lambda(s_i, t)) = \mu(s_i, t) + \epsilon(s_i) + \gamma(t) + \kappa(s_i, t) + \delta(s_i, t)$$

- One option in seperability

$$\Sigma_\kappa(\theta) = \Sigma_s \otimes \Sigma_t$$

- PDE Motivated Approach

$$\frac{\partial \eta}{\partial t} = \beta \frac{\partial^2 \eta}{\partial s} - \alpha \eta$$
$$\boldsymbol{\eta}_t = \boldsymbol{M}\boldsymbol{\eta}_{t-1} + \psi_t$$

Fitting Spatio-Temporal Models

- INLA (to me) is more straight forward
- 223 time points, 247 spatial locations
- 177 minutes to fit with $\kappa(s_i, t) \sim MVN(0, I)$, 266 to fit with $\Sigma_\kappa = \Sigma_{BYM} \otimes \Sigma_{RW1}$
- DIC prefers simpler model
- Forecasting done as missing data

Data Driven Processes - Moving from Latent Gaussian

- Structure placed on $\lambda(s_i, t)$ instead of $\log(\lambda(s_i, t))$
- Convolution of latent spatial and explicit temporal
- Can no longer fit in INLA

$$\lambda(s_i, t) = \kappa Z(s_i, t-1) + \exp(\mu(s_i, t) + \phi(s_i))$$

Data Driven Spatial Model with Hurdle

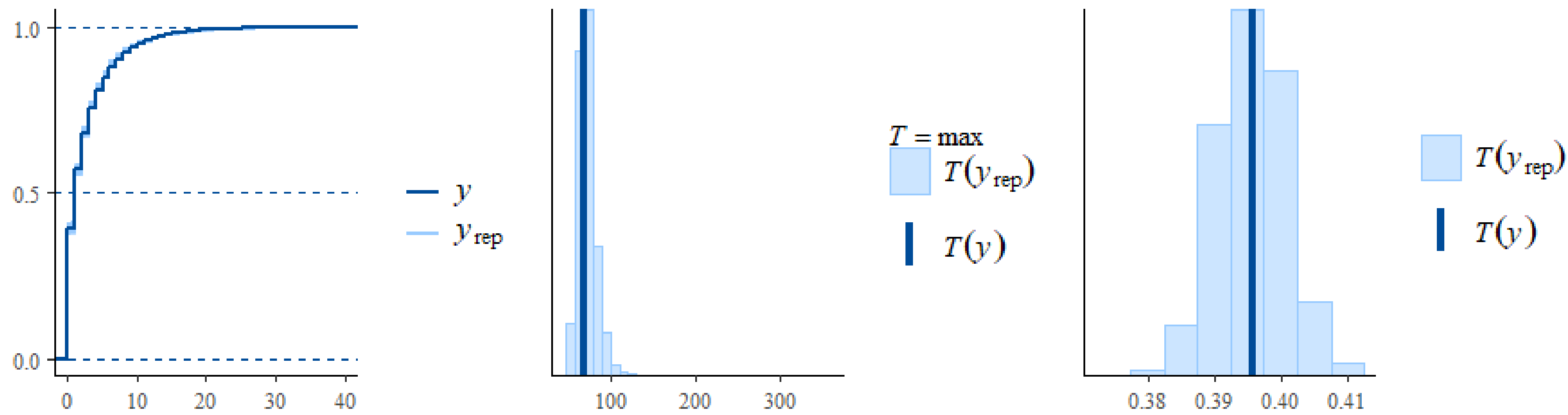
$$Pr(Z(s_i,t)=0)=\pi(s_i,t)$$

$$Pr(Z(s_i,t)>0)=(1-\pi(s_i,t))Po(Z(s_i,t|\lambda(s_i,t))1_{Z(s_i,t)>0}$$

$$\text{logit}(\pi(s_i,t))=\beta_0+\beta_1 x_{Pop(s_i)}$$

$$\log(\lambda(s_i,t))=\beta_0+\beta_1 x_{Dow(t)}+\phi(s_i)$$

Some preliminary results



ECDF, Maximum Value, Percent of Zeros

Interesting Lines of Research

- How do we capture longer temporal dynamics?
- What are practical differences between data driven processes and latent Gaussian driven processes?
- How should immune population be factored in? Cases divided by suceptible?
- How do we disentangle mobility from response variable?

Closing Thoughts

- Spatio-Temporal structure is primarily needed when large scale covariates fail to capture structure in data
- Things that are close together in time/space behave similarly
- Structure of covariance/precision matrix is necessary for computational reasons
- An appropriate statistical model should be able to replicate key characteristics of data

Some good resources

- “Statistics for Spatio-Temporal Data” - Cressie and Wikle
- “Spatial and Spatio-temporal Bayesian Models with R - INLA” - Blangiardo
- “Statistics for Spatial Data” - Cressie
- “Spatial and Spatio-Temporal Geostatistical Modeling and Kriging” - Montero, Fernandez-Aviles, Mateu